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Portfolio optimization problems in different risk measures using genetic algorithm

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ABSTRACT

This paper introduces a heuristic approach to portfolio optimization problems in different risk measures by employing genetic algorithm (GA) and compares its performance to mean–variance model in cardinality constrained efficient frontier. To achieve this objective, we collected three different risk measures based upon mean–variance by Markowitz; semi-variance, mean absolute deviation and variance with skewness. We show that these portfolio optimization problems can now be solved by genetic algorithm if mean–variance, semi-variance, mean absolute deviation and variance with skewness are used as the measures of risk. The robustness of our heuristic method is verified by three data sets collected from main financial markets. The empirical results also show that the investors should include only one third of total assets into the portfolio which outperforms than those contained more assets.

1. Introduction

Expect return and risk are the most important parameters with regard to portfolio optimization problems. One of the main contributions on this problem is by Markowitz (1952, 1991) who introduced mean–variance model, but the standard mean–variance model is based on assumption that investors are risk averse and the return of assets are normally distributed. Jia and Dyer (1996) noted that these conditions are rarely satisfied in practice. The mean–variance objective function may not be the best choice available to investors in terms of an appropriate risk measure. Furthermore, other risk measures may be more appropriate. From a practical point of view, real world investors have to face a lot of constraints in risk models: trading limitation, size of portfolio, etc. Such as constraints may be formed in a nonlinear mixed integer programming problem which is considerably more difficult to solve than the original model. Several researchers have attempted to find this problem by a variety of techniques, but exact solution methods fail to solve large-scale instances of the problem. Therefore, several researchers try to improve algorithms by using the state-of-the-art mathematical programming methodology to solving portfolio problems. The purpose of this paper is to show that portfolio optimization problems containing cardinality constrained efficient frontier can be successfully solved by the state-of-the-art genetic algorithms if we use the different risk measures such as mean–variance, semi-variance, mean absolute deviation and variance with skewness. We also show that practical portfolio optimization problems consisting of different numbers of assets drawn from three main markets stock indices can be solved by a genetic algorithm within a practical amount of time.

The remainder of this paper is organized as follows. Section 2 describes the portfolio optimization in the risk measures which we want to solve. In Section 3 investigates basic structure of genetic algorithm. Section 4, our proposed algorithm was introduced. Section 5 provides our computational results using C++ programming. It shows that cardinality constrained portfolio optimization problems can be solved in different risk measures without difficulty. Conclusion is given in Section 6.

2. Portfolio optimization in the risk measures

Portfolio is to deal with the problem of how to allocate wealth among several assets. The portfolio optimization problems have been one of the important research fields in modern risk management. In generally, an investor always prefers to have the return on their portfolio as large as possible. At the same time, he also wants to make the risk as small as possible. However, a high return always accompanied with a higher risk. Markowitz introduced the mean–variance model, which has been regarded as a quadratic programming problem. In spite of its popularity during the past, the mean–variance model is based upon the assumptions that an investor is risk averse and that either (i) the distribution of the rate of return is multivariate normal or (ii) the utility of the investor is a quadratic function of the rate of return. Unfortunately however, neither (i) nor (ii) holds in practice. It is now widely recognized that the real world portfolios do not follow a multivariate normal distribution. Many researchers once suggest that cannot blindly

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depend on mean–variance model. Therefore, there has been a tremendous amount of researches on improving this basic model both computationally and theoretically. Various risk measures such as semi-variance model, mean absolute deviation model and variance with skewness model have been proposed. Among them risk models were mathematically shown as below.

2.1. Mean–variance model

Markowitz was the first to apply variance or standard deviation as a measure of risk. He assumed that his classical formation is as follows:

$$\text{Minimize} \quad \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{1}$$

$$\text{Subject to} \quad \sum_{i=1}^{N} w_i \mu_i = R^* \tag{2}$$

$$\sum_{i=1}^{N} w_i = 1 \tag{3}$$

$$0 \leqslant w_i \leqslant 1, \quad i = 1,\ldots,N \tag{4}$$

where

N is the number of assets available;
w_i is the proportion ($0 \leqslant w_i \leqslant 1$) of the portfolio held in assets i ($i = 1,\ldots,N$);
μ_i is the expected return of asset i ($i = 1,\ldots,N$);
σ_{ij} is the covariance between assets i and j ($i = 1,\ldots,N$; $j = 1,\ldots,N$).

Eq. (1) minimizes the total variance (risk) associated with the portfolio while Eq. (2) ensures that the portfolio has an expected return of R^*. Eq. (3) ensures that the proportions add to one. In Eq. (4) the proportion held in each asset is between zero (minimum amount) and one (maximum amount). This formulation (Eqs. (1)–(4)) is a quadratic programming problem and nowadays it can be solved optimally using available software tool.

By solving the above optimization problem continuously with a different R^* each time, a set of efficient points is traced out. This efficient set called the efficient frontier and is a curve that lies between the global minimum risk portfolio and the maximum return portfolio. In other words, the portfolio selection problem is to find all the efficient portfolios along this frontier.

In order to enrich the model, we introduce a weighting parameter λ ($0 \leqslant \lambda \leqslant 1$) and consider:

$$\text{Minimize} \quad \lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{N} w_i \mu_i\right] \tag{5}$$

$$\text{Subject to} \quad \sum_{i=1}^{N} w_i = 1 \tag{6}$$

$$\sum_{i=1}^{N} z_i = K \tag{7}$$

$$\varepsilon_i z_i \leqslant w_i \leqslant \delta_i z_i, \quad i = 1,\ldots,N \tag{8}$$

$$z_i \in \{0,1\}, \quad i = 1,\ldots,N \tag{9}$$

where

K is the desired number of assets in the portfolio;
ε_i is the minimum proportion that must be held of asset i ($i = 1,\ldots,N$) if any of assets i is held;
δ_i is the maximum proportion that can be held of asset i ($i = 1,\ldots,N$) if any of assets i is held;
$z_i = 1$ if any of asset i ($i = 1,\ldots,N$) is held
 $= 0$ otherwise.

Eq. (5) the case $\lambda = 0$ represents maximum expected return and $\lambda = 1$ represents minimum risk. Values of λ satisfying $0 < \lambda < 1$ represent an explicit trade-off between risk and return, generating solutions between the two extremes $\lambda = 0$ and $\lambda = 1$. Eq. (6) ensures that the proportions add to one. Eq. (7) is assets desired number constraint. It ensures that exactly K assets are held. Eq. (8) constraints define lower and upper limits on the proportion of each asset which can be held in the portfolio. It ensures that if any of assets i is held ($z_i = 1$) its proportion w_i must lie between ε_i and δ_i, while if none of asset i is held ($z_i = 0$) its proportion w_i is zero. Eq. (9) is the integrality constraint. By a weighting parameter λ, we could use this program (Eqs. (5)–(9)) to trace out the cardinality constrained efficient frontier (CCEF) in an exactly analogous way. The use of heuristics for cardinality constrained portfolio optimization has been proposed and discussed by Chang, Meade, Beasley, and Sharaiha (2000).

2.2. Semi-variance model

Standard mean–variance model is based upon assumptions that an investor is risk averse and that the distribution of the rate of return is multivariate normal. This means that the variance component of the Markowitz quadratic objective function can be replaced by other risk functions such as semi-variance. With an asymmetric return distribution, the mean–variance approach leads to an unsatisfactory prediction of portfolio behavior. Markowitz indeed suggested that a model based on semi-variance would be preferable. Let:

T be such that we have observed historical values for stocks over the time period $0,1,2,\ldots,T$;
v_{it} be the value of one unit of stock i ($i = 1,\ldots,N$) at time t ($t = 0,\ldots,T$);
C_{cash} be the cash available to invest in the portfolio;
x_i be the number of units of stock i ($i = 1,\ldots,N$) that we choose to hold in the portfolio;
$z_i = 1$ if any of stock i ($i = 1,\ldots,N$) is held in the portfolio
 $= 0$ otherwise.

It is helpful when formulating the problem to introduce:

w_i is the proportion of C_{cash} that is invested at time T in stock i ($i = 1,\ldots,N$);
r_t is the single period continuous time return given by the portfolio at time t ($t = 1,\ldots,T$).

We get the values through the variables given previously:

$$w_i = v_{iT} x_i / C_{cash}, \quad i = 1,\ldots,N \tag{10}$$

$$r_t = \log_e\left(\sum_{i=1}^{N} v_{it} x_i \Big/ \sum_{i=1}^{N} v_{it-1} x_i\right), \quad t = 1,\ldots,T \tag{11}$$

Eq. (10) defines w_i to be the proportion of the portfolio associated with stock i at time T and Eq. (11) defines r_t to be the return on the portfolio (since the total value of the portfolio at time t is $\sum_{i=1}^{N} v_{it} x_i$). Then the constraints associated with discrete time portfolio optimization problem are

$$\sum_{i=1}^{N} z_i = K \tag{12}$$

$$\varepsilon_i z_i \leqslant v_{iT} x_i / C_{cash} \leqslant \delta_i z_i, \quad i = 1,\ldots,N \tag{13}$$

$$\sum_{i=1}^{N} v_{iT} x_i = C_{cash} \tag{14}$$

$$x_i \geqslant 0, \quad i = 1,\ldots,N \tag{15}$$

$$z_i \in \{0,1\}, \quad i = 1,\ldots,N \tag{16}$$