Policy effects in hyperbolic vs. exponential models of consumption and retirement ☆

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ABSTRACT

This paper constructs a structural retirement model with hyperbolic preferences and uses it to estimate the effect of several potential Social Security policy changes. Estimated effects of policies are compared using two models, one with hyperbolic preferences and one with standard exponential preferences. Sophisticated hyperbolic discounters may accumulate substantial amounts of wealth for retirement. We find it is frequently difficult to distinguish empirically between models with the two types of preferences on the basis of asset accumulation paths or consumption paths around the period of retirement. Simulations suggest that, despite the much higher initial time preference rate, individuals with hyperbolic preferences may actually value a real annuity more than individuals with exponential preferences who have accumulated roughly equal amounts of assets. This appears to be especially true for individuals with relatively high time preference rates or who have low assets for whatever reason. This affects the tradeoff between current benefits and future benefits on which many of the retirement incentives of the Social Security system rest.

Simulations involving increasing the early entitlement age and increasing the delayed retirement credit do not show a great deal of difference whether exponential or hyperbolic preferences are used, but simulations for eliminating the earnings test show a non-trivially greater effect when exponential preferences are used.

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1. Introduction

The impending advance of the baby boom generation into their retirement years, and perhaps even more importantly, the decline in the birth rate, have created strains on the Social Security system (Goss, 2010). As a result, increasing attention is being paid to measures that might encourage workers to stay on the job longer, contribute more to the system, and hence relieve some of the strain.[1] Some of the proposals which have been mentioned have been increases in the early entitlement age, increases in the normal retirement age (which amounts to a reduction in benefits), and elimination of the earnings test, among others. An important goal of current research in this field, including the present paper, is to gauge the likely success of such measures in increasing the retirement age.

This paper brings together two important strands of the recent literature on retirement and savings. One of these strands is the development of increasingly realistic structural models of retirement and saving. Empirical structural models of retirement began as relatively simple decisions as to the optimal time to retire.[2] Over time, advances in computational capacity have allowed these early models to evolve into far richer models involving more nuanced decision sets and more elements of uncertainty.[3] The decision to retire now includes the possibility of partial retirement as well as the possibility of returning to work after a period of retirement.[4] The stochastic environment includes not only mortality but also uncertainty as to the returns to assets and the degree to which an individual will find retirement enjoyable after the fact.[5] Other authors introduce uncertainty in wages and unpredictable health and health care

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[1] The U.S. Senate's Special Committee on Aging (2010) discusses a number of options for Social Security reform.

[2] See, for instance, Gustman and Steinmeier (1986) as an early example.

[3] See, for instance, Rust and Phelan (1997), French and Jones (2004), French (2005), Scholz et al. (2006) and Van der Klaauw and Wolpin (2006).

[4] As an example, see Gustman and Steinmeier (2008b).

[5] See Gustman and Steinmeier (2002).

expenditures. The result has been that the structural models of retirement and saving have increasingly been able to encompass the major elements affecting the retirement decision.

A second strand of literature is the analysis of so-called "hyperbolic" preferences. These preferences have been introduced in recent years to reflect the fact that many individuals place a heavy weight on current consumption, but do not distinguish incremental years in the future quite so much. Proponents of hyperbolic preferences (Laibson, 1997) point to a wide range of phenomena which hyperbolic preferences help to explain, such as simultaneously having high-interest credit card debt and low-interest individual retirement accounts. The hyperbolic preferences used in this paper are actually what are called "quasi-hyperbolic" preferences, with a high discount rate between the current period and the next period and lower discount rates between successive future periods.

The literature on hyperbolic preferences focuses mainly on consumption and saving behavior. While saving for retirement may be an important element of this behavior, the actual retirement date is largely taken as fixed. On the other hand, the structural retirement models almost uniformly assume exponential preferences, where a uniform time preference rate, perhaps differing among individuals, is applied to all future periods.

The purpose of this paper is to construct a structural retirement model that can encompass either exponential or hyperbolic preferences. We estimate the model twice, once using exponential preferences and once using hyperbolic preferences. Using these estimates, we can examine the differences in retirement and consumption outcomes between the two models, and estimate and compare the effects of several potential policy changes that would be implied by the two models. Such comparisons can indicate the sensitivity of the estimated structural models to assumptions regarding the type of preferences.

The next section looks at several properties of hyperbolic models and examines the degree to which it is possible to differentiate hyperbolic preferences from exponential preferences in the data. Section 3 specifies in more detail a structural model which can encompass either exponential or hyperbolic preferences, and Section 4 discusses the stochastic specification and estimation of the model. Section 5 analyzes several simulations with the estimated model to examine what difference the choice of preferences makes to the estimated effects of potential policy changes. Concluding observations are contained in the last section.

2. Properties of hyperbolic models

In this section we will consider some of the properties of models with hyperbolic preferences and compare them to models with exponential preferences. With exponential preferences, expected utility at any period t can be expressed as

$$E_t \, U = E_t \left[\sum_{i=t}^{T} \left(\frac{1}{1+\rho} \right)^{i-t} u(C_i) \right]$$

where $u(C_i)$ is the usual increasing but with diminishing marginal returns within-period utility function, and ρ is the time preference rate. The term $\frac{1}{1+\rho}$ is the discount rate. For hyperbolic preferences, expected utility is

$$E_t \, U = u(C_t) + \frac{1}{1+\rho_1} \, E_t \left[\sum_{i=t+1}^{T} \left(\frac{1}{1+\rho_2} \right)^{i-(t+1)} u(C_i) \right]$$

In the hyperbolic model, the time preference rate between t and t + 1 is ρ_1, and between any two adjacent time periods after t + 1 the time preference rate is ρ_2. In the presumed case where $\rho_1 > \rho_2$, the individual weighs the current period more heavily than future periods, but does not distinguish quite as much among future periods. Exponential

preferences may be viewed as a subset of hyperbolic preferences with $\rho_1 = \rho_2$.

Theorists distinguish among two variants of the hyperbolic model. In the first variant, the individual assumes that whatever consumption stream he chooses at time t will be followed in successive periods. This means that he thinks that at time t + 1, the time preference rate between t + 1 and t + 2 will still be ρ_2, despite the fact that at time t, the time preference rate between t and t + 1 is ρ_1. Since this is unlikely to be the case, such an individual is commonly called "naïve." The alternative case is that the individual knows that although at time t the time preference rate between t + 1 and t + 2 is ρ_2, when the individual reaches time t + 1 the time preference rate between these same two periods will be ρ_1. That is, he knows that his preferences are inconsistent across time and takes this into account. The literature refers to such individuals as "sophisticated."[6]

In this paper we will deal with individuals with sophisticated hyperbolic preferences. Naïve hyperbolic individuals would not be able to accumulate any significant amount of wealth, contrary to what is frequently observed. Though they would recognize the need to save for retirement, they would always find it advantageous to wait until next year to start saving. Sophisticated hyperbolic individuals recognize this problem, and they are able to save significant amounts despite having a high time preference for the current period over future periods.

2.1. Exponential vs. hyperbolic models for a representative individual

Consider next the difficulties when attempting to distinguish exponential vs. hyperbolic preferences among individuals in household surveys such as the Health and Retirement Study. Here we consider a multiperiod model which begins at age 25. The model incorporates survival tables in calculating future expected utilities, up to a maximum age of 100. This representative individual is assumed to have a steady income from earnings of $25,000 (in 1992 dollars) from age 25 until he retires at age 62. The wife, who is 2 years younger, is assumed to have a steady income of $15,000 from age 25 until age 62, at which point she also retires. The husband's PIA is assumed to be around $10,000 and the wife's PIA is assumed to be around $7000. To keep things simple, neither spouse is assumed to have a pension. These numbers are fairly close to the median amounts for the original HRS cohort.

In Fig. 1 we look at the asset trajectory of this individual. The solid line in Fig. 1 details the levels of assets for the individual if he had hyperbolic preferences with a discount factor of 0.67 ($\rho_1 = 0.50$) for the first period and an additional discount factor of 0.98 ($\rho_2 = 0.02$) for each period beyond the first period. As has been previously noticed in the literature (Harris and Laibson, 2002), individuals with hyperbolic preferences may accumulate a substantial amount of wealth as long as those preferences are "sophisticated," meaning that in each time interval the individual realizes that his future preferences, and in particular the low discount rate for periods subsequent to the next period, will not be the same as his current preferences. The dashed line in the figure details the levels of assets for the individual if he had exponential preferences with a discount factor of 0.958 ($\rho = 0.041$), which would yield approximately the same amount of assets at age 62 as would the hyperbolic preferences.

For individuals in the age range 50 to 65, which is typical of the age range for which we observe individuals in the HRS, the graph suggests

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[6] Hyperbolic discounting has been used to explain the failure to save and anomalies in saving behavior. An extension by Diamond and Koszegi (2003) jointly analyzes retirement and saving behavior. This work focuses on sophisticated hyperbolic discounters and the actions taken by early selves to influence the behavior of later selves. Diamond and Koszegi discuss cases where, to augment own welfare in later years, one might either undersave to discourage later selves from too early a retirement, or may subsidize an undesired early retirement through additional early savings, even though early selves favor later retirement.