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On optimal portfolio trading strategies for an investor facing transactions costs in a continuous trading market

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Abstract

Modern asset pricing theory generally assumes frictionless trading. Under this assumption, an investor would revise his portfolio holdings at every date on which he could trade. However, in models where an investor faces financial market frictions such as transactions costs, the portfolio is optimally rebalanced less frequently. This paper examines the portfolio trading problem for an investor who faces transactions costs and short sales constraints in a continuous time economy with general specifications of ask and bid prices. Our principal results state that the existence of the optimal trading strategy and solution to the investor problem implies the existence of two supermartingales whose ratio is bounded by the ask and bid prices and we can identify supporting prices which, in an economy with no transactions costs, would yield the trading strategy and optimal solution of the original economy. This leads to explicit representations of the value function for utility functions commonly analyzed in financial economics.

Keywords: Portfolio; Investor; Trading market

1. Introduction

Modern asset pricing theory generally assumes frictionless trading. Under this assumption, an investor would revise his portfolio holdings at every date on which

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he could trade. However, in the real world, people revise their asset holdings much less frequently. Some insight into resolving this puzzle is gained by Constantinides (1979), who shows that an investor facing transactions costs revises his portfolio less frequently. Specifically, the state space can be divided into three regions determined by current portfolio holdings: buy, sell and do not transact. For this reason, there has been a great deal of interest in models with transactions costs. This model is extended by Constantinides (1986), Davis and Norman (1990), and Dumas and Luciano (1991) to the continuous time case for power utility functions and proportional transactions costs.

In all of these approaches, prices are taken as lognormally distributed. However, as Dumas and Luciano (1991) carefully note, we do not know whether these prices could ever arise as equilibrium prices. This suggests the need to investigate the issue and to do so requires a more general model of the consumption–investment problem.

There are two approaches to solving consumption–investment problems in frictionless economies. The first, pioneered by Merton (1971) for continuous trading, involves using Hamilton–Bellman techniques. The second approach stems from the observation by Harrison and Kreps (1979) that arbitrage free price systems admit an equivalent probability measure under which discounted asset prices become martingales. This observation is exploited initially in Pliska (1986), Karatzas et al. (1987) and Cox and Huang (1989) to solve the consumption investment problem in the complete market case, and in He and Pearson (1991) and Karatzas et al. (1991) to solve the consumption investment problem for the incomplete market case.

Recently, Jouini and Kallal (1995) and Jouini (1997) extend the analysis of Harrison and Kreps (1979) to the case where investors incur transactions costs and must obey short sales constraints when they trade assets. They show that absence of arbitrage opportunities in these types of markets implies the existence of supermartingales with certain properties. However, in their analysis, they restrict the trading strategies to simple ones, that is, strategies which only change at a finite number of predetermined times, in order to rule out doubling strategies. Moreover, their analysis effectively assumes terminal wealth to be in L^2 and preferences to be norm continuous, which rules out many commonly used utility functions. It would be interesting to evaluate the robustness of these results if agents are permitted to trade continuously using a class of preferences which may not satisfy the assumptions in Harrison and Kreps (1979).

This paper uses ‘martingale’ methods to characterize the portfolio trading problem when an investor faces transactions costs in a continuous trading financial market. Our model consists of an investor who wishes to trade assets in such a way as to maximize the expected utility of terminal wealth. We model transactions costs by assuming the investor buys one of the assets at an ask price and sells this asset at a bid price. The ask and bid prices can be specified in a very general way. Our analysis is similar to that of Duffie and Skiadas (1994). Indeed, our most