Optimal portfolio policies with borrowing and shortsale constraints

Lucie Teplá*

INSEAD, Boulevard de Constance, 77305 Fontainebleau cedex, France

Accepted 30 November 1999

Abstract

We characterize optimal intertemporal portfolio policies for investors with CRRA utility facing either a borrowing constraint, or shortsale restrictions, or both. The optimal constrained portfolios are identified as optimal unconstrained portfolios for a higher riskless rate, or for a subset of the risky assets, or for a combination of the two settings. Our characterization is based on duality results in Cvitanić and Karatzas (1992, Annals of Applied Probability 2, 767–818) for optimal portfolio investment when portfolio values are more generally constrained to a closed, convex, nonempty subset of \( \mathbb{R}^n \). © 2000 Elsevier Science B.V. All rights reserved.

JEL classification: G11

Keywords: Optimal investment; Borrowing constraints; Shortsale constraints

1. Introduction

This paper characterizes optimal intertemporal portfolio policies for CRRA-utility investors facing either a borrowing limit on the total wealth invested in

---

*Tel.: + 33-1-60-72-44-85; fax: + 33-1-60-72-40-45.
E-mail address: lucie.tepla@insead.fr (L. Teplá).

The author wishes to thank two anonymous referees and especially the editor, Michaël Selby, for useful comments and discussion.

0165-1889/00/$ - see front matter © 2000 Elsevier Science B.V. All rights reserved.

PII: S 0 1 6 5 - 1 8 8 9 ( 9 9 ) 0 0 0 8 9 - 5
the risky assets, or shortsale restrictions on all risky assets, or both. The characterization is based on the first-order conditions to a minimization problem identified by Cvitanić and Karatzas (1992) as underlying the dual formulation of the optimal portfolio investment problem when portfolio values are more generally constrained to a closed, convex, nonempty subset of $\mathbb{R}^n$ (when there are $n$ risky assets). In each setting, the optimal constrained portfolio is identified as an optimal ‘unconstrained’ portfolio. Specifically, with borrowing constraints only, CRRA-utility investors act as if unconstrained but facing a higher interest rate. With shortsale constraints only, these investors act as if unconstrained when investing only in a subset of the risky assets. With borrowing and shortsale constraints, both effects obtain. Specifically, the optimal portfolio is equivalent to the optimal borrowing-constrained-only portfolio for a subset of the risky assets, and thus to the optimal unconstrained investment in these assets at a higher interest rate.

Results closely related to a number of those derived here in a dynamic setting have previously been identified as holding in a one-period, mean-variance or Markowitz framework. Black (1972) establishes that an investor who cannot borrow at all chooses a tangency portfolio corresponding to a higher interest rate. Brennan (1971) considers the setting in which the investor can borrow without limit, but faces a borrowing rate which is greater than the lending rate. The optimal portfolio is again equivalent to a tangency portfolio, in this case corresponding to one of three possible ‘risk-free’ rates. Separately, Lintner (1965) identifies the optimal shortsale-constrained Markowitz portfolio as the optimal unconstrained portfolio for a subset of the risky securities. The fact that we obtain very similar results for CRRA-utility investors in the dynamic setting is not entirely unexpected given that it is well-known that, for the model we consider, these investors’ optimal unconstrained portfolios are instantaneously mean-variance efficient. Nonetheless, to date, some of these results, particularly those concerning shortsale constraints, have been missing from the continuous-time literature. Grossman and Vila (1992), using a stochastic dynamic programming approach, study the optimal intertemporal portfolio policies of a borrowing-constrained power-utility investor in the standard Merton (constant-coefficient) setting. Rather than restricting investment in the risky assets to be less than some constant proportion of wealth, as we do here, Grossman and Vila consider the effects of a borrowing limit which is affine in wealth. Because their model features only one risky asset, it does not identify how a borrowing

---

1 The relevant rate is either the borrowing rate, the lending rate, or a rate at which the optimal investment results in neither borrowing nor lending.

2 See, for example, Merton (1990, pp. 170–171). Here we assume a deterministic investment opportunity set, so there is no hedging demand.

3 This feature leads the investor to alter his optimal portfolio holdings even when the constraint is not binding (see Grossman and Vila, 1992).
دریافت فوری متن کامل مقاله

امکان دانلود نسخه تمام متن مقالات انگلیسی
امکان دانلود نسخه ترجمه شده مقالات
پذیرش سفارش ترجمه تخصصی
امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
امکان دانلود رایگان ۲ صفحه اول هر مقاله
امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
دانلود فوری مقاله پس از پرداخت آنلاین
پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات