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High accuracy estimation of multi-frequency signal parameters by improved phase linear regression

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Abstract

An improved phase regression approach for estimating the parameters of a multi-frequency signal from discrete samples corrupted by additive noise is presented. It efficiently estimates the signal frequency and phase by linear regression on the phase spectra of segmented signal blocks, and the signal amplitude directly from the discrete-time Fourier transform of the window function. The techniques of weighted spectral lines averaging and overlapped signal segmenting are introduced to improve the estimation accuracy. The expressions of the estimator variances are derived, and shown to almost reach the Cramer–Rao bounds. Numerical simulations are given to confirm the validity of the presented approach.

Keywords: Spectral analysis; Parameter estimation; Discrete Fourier transform; Linear regression; Statistical analysis

1. Introduction

A typical problem in harmonic analysis and electronic measurement is the estimation of the parameters, i.e. amplitude, frequency and phase, of each component of a multi-frequency signal from discrete-time observations [1]. Despite the development of parametric analysis [2], the windowed discrete Fourier transform (DFT) remains the most useful tool in this field because of its quick and easy implementation through fast Fourier transform (FFT) and less sensitivity to algorithm design parameters. Due to the spectrum granularity effect (also known as leakage and picket-fence effects) caused by a finite number of processed samples, the estimates resulting from the DFT are normally different from the real ones. For instance, the maximum error of the phase estimate may amount to $\pm 90^\circ$, regardless of the window function and frequency resolution adopted. It is well known that there is no spectral leakage in the DFT of a finite synchronously sampled periodic signal sequence, in which an integer multiple of periods is measured. However, in practical situations, it is impossible for the sampling procedure to be exactly synchronized with every component of a general multi-frequency signal. Therefore, to overcome the spectrum granularity effect is of great significance for practical use. Moreover, during the real measurement process, various uncontrollable factors manifest themselves with uncertainties (noise) in the collected data. Thus, if one is interested in high-precision applications, the specification of the noise effect has to be taken into account.

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Traditionally, flat top windows are utilized to reproduce the correct amplitude of each signal component. However, this technique could not eliminate the errors of frequency and phase estimates caused by spectrum granularity effect at all. The approach given in [3], which aliases the time domain signal to suppress the frequency domain leakage, still has this drawback. In essence, it corresponds to only calculating the kMth ($k = 0, \ldots, N-1$) spectral samples when processing a signal sequence of length MN ordered from 0 to $MN-1$. Following the simple idea that selecting a shorter window that has a broader spectral main-lobe would reduce the influence of the picket-fence effect, a procedure was presented to estimate the proper window length for a given limit of amplitude error [4]; the resulting phase estimate is still very poor. By exploiting the characteristics of the energy distribution of the signal in frequency domain, a simple approach was proposed for accurate real-time signal parameter estimation [5]. However, it is highly sensitive to the noise interference. The (almost) maximum likelihood estimation method, which combines DFT with discrete-time Fourier transform to locate the peaks of the amplitude spectrum, was found to be very time-consuming [6,7], although it provides the most accurate estimates in the presence of noise. With help of interpolation technique, strategies with different expressions [1,8–14] were introduced for accurate recovery of the signal parameters. For some typical windows, there is an explicit solution available, while for other complicated windows, an iterative procedure is needed. The noise influence on the accuracy of this method was studied by Schoukens et al. [15] qualitatively, and by Offelli and Petri [16] quantitatively. A frequency domain least-squares method with good statistical performance in amplitude estimation was proposed by Carbone [17]. A simplified linear model was adopted due to the complexity of the comprehensive complex-valued model. Later, with two weak restrictions on the window function used, a concise nonlinear least-squares based parameter estimation model was presented, and a two-step iterative algorithm was developed to efficiently estimate the signal parameters [18]. Based on the quantitative relationship between the phases resulting from the DFTs of the original signal and the shifted signal with the same or different length, the phase-difference method was initially proposed only for estimating the frequency of a monochromatic signal [19], and was fully developed afterwards [20,21]. It requires twice calculation of DFT. From different point of views, similar ideas were explored by Hanrahan [22], and Dai and Gretsch [23]. Statistical performance analysis showed that this method had superior accuracy over the interpolation method [21]. Also, Mcmahon and Barrett gave a frequency estimator that determines the frequency by locating the peak of the discrete-time Fourier transform of the DFTs of the segmented signal blocks [24]. The maximum likelihood estimator and the phase-difference estimator were shown to be two special cases of this one. As for periodic signals, two specific techniques were developed to improve the accuracy of the spectral analysis. One is to modify the actual sampled sequence to be an ideal sample sequence that is synchronized with the original periodic signal [25]. The other is to improve the estimate of the period length by minimizing a cost function [26].

In [27], Tretter proposed a time domain method to estimate the frequency of a noisy sinusoid by linear regression on the instantaneous phases of the complex analytical signal; the estimator variance reaches the Cramer–Rao bound. Kay further gave an improvement to overcome some drawbacks caused by the circular nature of the phase [28]. These two approaches are only applicable to single tone with very high signal-to-noise ratio (SNR). Umesh and Nelson noticed the relationship between this kind of method and the phase-difference method, and developed a frequency domain approach that can estimate the sinusoidal frequency at low SNR [29]. The major drawbacks are that it can only estimate the frequency parameter of a monochromatic signal, and that the circular nature of the phase spectrum used in the algorithm is thoroughly overlooked. Therefore, it is unacceptable for practical use. Recently, the phase regression approach [30], which efficiently estimates the signal frequency and phase by linear regression on the short-time phase spectra, was proposed as the extension of phase-difference approach and Umesh and Nelson's approach. In this paper, the technique of overlapped signal segmenting will be introduced to further improve its estimation accuracy with the help of repetitive use of overlapping data. The results are exciting; the estimator variances are shown to almost reach the Cramer–Rao bounds.

The remainder of this paper is organized as follows. In Section 2, the frequency domain additive phase/amplitude noise model is briefly introduced. In Section 3, the improved phase regression