The one-year non-life insurance risk

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\textbf{A B S T R A C T}

A major part of the literature on non-life insurance reserve risk has been devoted to the \textit{ultimo} risk, the risk in the full run-off of the liabilities. This is in contrast to the short time horizon in internal risk models at insurance companies, and the one-year risk perspective taken in the Solvency II project of the European Community.

This paper aims at clarifying the one-year risk concept and describing simulation approaches, in particular for the one-year reserve risk. We also discuss the one-year premium risk and its relation to the premium reserve.

Finally, we initiate a discussion on the role of risk margins and discounting for the reserve and premium risk, with focus on the Cost-of-Capital method. We show that risk margins do not affect the reserve risk and show how reserve duration can be used for easy calculation of risk margins.\textsuperscript{1}

\textsuperscript{1} Any opinions expressed in this paper are the authors’ own, and do not constitute policy of our employers.

In the actuarial literature, on the other hand, reserve risk has traditionally been discussed in terms of the risk that the estimated reserves will not be able to cover the claims payment during the full run-off of today’s liabilities, which may be a period of several decades; we call this the \textit{ultimo} risk. If \( R_0 \) is the reserve estimate at the beginning of the year and \( C_{\infty} \) are the payments over the entire run-off period, this risk is measured by studying the probability distribution of \( R_0 - C_{\infty} \). This is the approach of the so-called \textit{stochastic claims reserving} which has been developed in the actuarial literature over the last two decades, by Mack (1993), England and Verrall (2002) and many others.

Considering this background, it may not be surprising that it is noted in a study from the mutual insurers organization AISAM-ACME (2007), that “Only a few members were aware of the inconsistency between their assessment on the ultimate costs and the Solvency II framework which uses a one-year horizon”. Furthermore, the study shows that for long-tailed business, the ultimo (or as it is called there: full run-off) approach gives risk estimates that are 2 to 3 times higher than those for the one-year result. We conclude that it is both necessary and important to clarify the difference between the one-year and ultimo perspective.

In Section 6 of Dhaene et al. (2008), an approximate rule is given indicating that a one-year certainty level of 99.5% corresponds to a 40-year full run-off level of 81.8%. If we suppose that both risks are normally distributed, this can be seen to correspond to a volatility 2.8 times larger for the full run-off case, which is in line with the AISAM-ACME study. (On the other hand, Dhaene et al., at the end of Section 5, discuss an unclear point in the methodological description of the AISAM-ACME study that might explain some of the differences.)
On the methodological side, a few papers on the one-year reserve risk have recently appeared in the literature, see Wütrich et al. (2009) or Merz and Wütrich (2008). In the special case of a pure Chain Ladder estimate, they give analytic formulae for the mean squared error of prediction of the one-year result under an extension of the classic Mack (1993) model for the ultimo result. Wütrich and Bühmann (in press) model the one-year risk when claims reserves are discounted.

Our first aim is to help in clarifying the methodological issues for the one-year approach to reserve risk, and in particular to describe a general simulation approach to the problem. A special case of this approach is the bootstrap methods in the context of Dynamic Financial Analysis which are implemented in some commercial software; cf. Björkwall et al. (in press).

In Section 3 we turn to the one-year perspective on premium risk, followed by a discussion in Section 4 on the role of the risk margin for premium and reserve risk. To the best of our knowledge, these issues have not been discussed in the literature before.

2. The one-year reserve risk

We will think of the one-year risk as being computed on January 1 of the new year. For \( t = 1, 2, 3, \ldots \), let \( C_t \) be the amount paid during year \( t \), with the new year denoted by \( t = 1 \). In order to separate between premium and reserve risk we will split \( C_t \) into \( \bar{C}_t \), paid for risk years previous to \( t \), and \( \tilde{C}_t \), paid for the new risk year \( t \). Similarly, the closing claims reserve by the end of year \( t \), \( R_t \) is split into \( \bar{R}_t \) for historical risk years and \( \tilde{R}_t \) for claims from the new year \( t \). Note that

\[
C_t = \bar{C}_t + \tilde{C}_t \quad \text{and} \quad R_t = \bar{R}_t + \tilde{R}_t.
\]

We extend the notation to include \( R_0 \), the opening claims reserve for year \( t = 1 \). The reserve risk over a one-year time horizon is the risk in the claims development result (CDR), also called the (one-year) run-off result, which is

\[
\bar{T} = R_0 - \bar{C}_1 - \bar{R}_1.
\]

Note that \( \bar{T} \) is also the difference between the estimate of the ultimate cost for these risk years at times 0 and 1. The one-year reserve risk is captured by the probability distribution of \( \bar{T} \). This is in contrast to the ultimate or full run-off risk, which was described above as the risk in \( R_0 - C_{\infty} \).

From the distribution of \( \bar{T} \) we can compute any risk measure of interest. Here we will focus on the risk measure used in Solvency II: the Value-at-Risk at the level 99.5%, see EU Commission (2007, Article 100). Let VaR(\( L \)) denote this risk measure for a loss \( L \), i.e. the 99.5% quantile of the loss distribution: if \( L \) is continuous then VaR(\( L \)) is the solution to \( Pr(L \leq \text{VaR}(L)) = 99.5\% \). Here and below we assume that \( E(L) = 0 \). In the notation of Dowd (1998, p. 43) we say that we use the relative VaR.

2.1. Simulating the one-year reserve risk

In practical situations, unless normal distributions are used, simulation methods will often be the only possibility to assess the reserve risk. Here we will describe the steps in simulating the one-year reserve risk, in the case when we neither use discounting nor add a risk margin to the reserves. This simulation algorithm is by no means new, but, to the best of our knowledge, it has not been discussed in the literature before, except for a short description in a bootstrap context in a preprint of Björkwall et al. (in press).

2.1.1. Step 1: Best estimate of the opening reserve

Available is the actuary’s best estimate of the outstanding claims at the beginning of the year, the opening reserve \( R_0 \), which is treated as deterministic here since it is based on values in \( D_0 \). In the Solvency II framework, reserves shall be the sum of a best estimate and a risk margin, where the best estimate is the expected present value of future cash-flows. We postpone a discussion of discounting and risk margins to Section 4, so in the present section the claims reserve is just the expected future cash-flows.

For a discussion on how to calculate the best estimate, see Groupe Consultatif (2008). We agree with this paper in that no detailed rules can be given for the choice of methods; here we will merely assume that the best estimate was computed by a documented algorithm \( A_1 \) that could be repeated in the simulations. For a long-tailed business such an algorithm might, e.g., be to use a development factor method (such as the Chain Ladder) on paid claims, with the factors smoothed and extended beyond the observation years by some regression model; then a Generalized Cape Cod (GCC) method may be used to stabilize the latest years, while the earliest years reserves might be adjusted somehow by the claims incurred. For a description of the GCC, see Struzzieri and Hussian (1998).

In our opinion, the quite popular Bornhuetter–Ferguson (BF) method does not fulfill the requirement of being algorithmic. The reason is that it uses some a priori loss ratios, whose calculation is outside the method. In practice, these loss ratios will probably be estimated from loss data and in order for BF to be algorithmic, this estimation should be made explicit. The GCC could be seen as a way of making BF algorithmic and is hence preferred here.

The use of a repeatable algorithm \( A_1 \) might be considered good practice, leaving as little as possible to the actuary’s subjective judgement on individual figures. If such judgement is still necessary, we have to find an approximate algorithm \( A_1 \), capturing the main features of the best estimate, for use in the simulation, Step 3.

The AISAM-ACME (2007) study distinguishes between the shock period, which is the one projection year when adverse events occur, and the effect period, which is the full length of the run-off of the liabilities. The direct effect in the shock period is captured by \( \bar{C}_1 \), while the variation in \( \tilde{R}_1 \) relates to the effect period. Hence the one-year risk is, to some extent, affected also by the risk during the entire life-span of the liabilities, but not as much as the ultimo risk is.

Note. We tacitly assume that an expenses reserve is included in the claims reserve above. The question of how this should affect the total reserve risk is outside the scope of this paper. □

In risk models, the insurance portfolio is divided into more or less homogeneous segments, e.g. lines of business (LoB). To be able to calculate the reserve risk for a segment and combine it with other risks to form the total risk of the insurer, we need the probability distribution of the CDR \( \bar{T} \) for this segment. Unless otherwise stated, the discussion below applies to the lowest level in the segmentation chosen in the risk model at hand.
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