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Optimal strategies for hedging portfolios of unit-linked life insurance contracts with minimum death guarantee

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ABSTRACT

In this paper, we are interested in hedging strategies which allow the insurer to reduce the risk to their portfolio of unit-linked life insurance contracts with minimum death guarantee. Hedging strategies are developed in the Black and Scholes model and in the Merton jump–diffusion model. According to the new frameworks (IFRS, Solvency II and MCEV), risk premium is integrated into our valuations. We will study the optimality of hedging strategies by comparing risk indicators (Expected loss, volatility, VaR and CTE) in relation to transaction costs and costs generated by the re-hedging error. We will analyze the robustness of hedging strategies by stress-testing the effect of a sharp rise in future mortality rates and a severe depreciation in the price of the underlying asset.

RÉSUMÉ

Dans ce papier, nous nous intéressons à la couverture des contrats en unités de compte avec garanties décès. Nous présentons des stratégies de couverture opérationnelles permettant de réduire de façon significative les coûts futurs liés à ce type de contrats. Suivant les recommandations des nouveaux référentiels (IFRS, Solvabilité 2 et MCEV), la prime de risque est introduite dans les évaluations. L'optimalité des stratégies est constatée au moyen de la comparaison des indicateurs de risque (Pertes espérée, écart type, VaR, CTE et perte Maximale) des stratégies dans le modèle standard de Black–Scholes et dans le modèle à sauts de Merton. Nous analysons la robustesse des stratégies à une hausse brutale de la mortalité future et à une forte dépréciation du prix de l'actif sous-jacent.

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1. Introduction

The new frameworks (Accountant: IFRS/IAS, Prudential: Solvency II, and financial communication: Market Consistent Embedded Value) encourage insurance companies to adopt an economic approach when evaluating their liabilities (Thérond, 2007). On this subject, the concept of "Fair Value" is fundamental. The fair value of an asset or a liability is the amount for which two interested and informed parties would exchange this asset or this liability. Fair values are usually taken to mean arbitrage-free values, or values consistent with pricing in efficient markets. The arbitrage-free valuation of an item is one which makes it impossible to guarantee riskless profits by buying or selling the item. This leads to the concept that if two portfolios have identical cash flows, and the portfolios can be priced in an efficient market, then the two portfolios will have the same price. Otherwise, an investor could sell one portfolio, buy the other and make free money. The fair value is therefore the price that the market naturally assigns to any tradable asset.

Risk-neutral valuation produces the fair value of any liability. As noted by Milliman Consultants and Actuaries (2005) the main reason for using risk-neutral or fair valuations is because they represent the objective market cost of purchasing a replicating portfolio in terms of the liability, thus ensuring that the company will have sufficient resources to meet the liability over all possible market movements. Risk-neutral valuation effectively translates the risky, market-dependent costs of the guarantee into a fixed cost item for the insurance company.

Thus, using the logic of fair valuation, purchasing a replicating portfolio is essential in the evaluation of liabilities. Accordingly, in the case of unit-linked life insurance for example, Frantz et al.

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(2003) showed that fair valuation is only valid if the underlying hedging is actually applied.[1] In such contracts, the return obtained by the policyholders on their savings is linked to some financial asset, and in this way it is the policyholder who supports the risk of the investment. The investment can be made on one asset or on a portfolio of assets, and various types of guarantees can be added to the pure unit-linked contract. In our study, we shall concentrate on the minimum death benefit guarantee. In this case, the insurer's liability in the case of the death of the policyholder will be $\max(K, V) = V + [K - V]^+$, where V is the value of the unit-linked contract and K is the guarantee. If $V < K$ then the insurer will pay the additional amount $K - V$. It therefore stands that the risk related to these contracts is real. However, this risk is often underestimated by the insurance companies, which then expose themselves to massive losses connected to a market in strong decline.

Frantz et al. (2003) analyzed delta hedging within the framework of Black and Scholes' model (1973). The Black and Scholes model assumes that the return process is continuous, distributed according to a normal distribution, and that its volatility is constant during time. However, the empirical reports show that none of these assumptions are always true when applied to the markets, as shown by the works of Cont (2001). Moreover, the classic valuation of unit-linked contracts assumes a perfect mutualisation of the deaths in the insurance portfolio. It therefore follows logically that we can wonder about the effectiveness of the insurer setting up a hedging strategy in order to protect from abnormally high death rates in the portfolio in the future.

Moreover, other hedging strategies exist. Hedging strategies which we can develop come primarily from the methods used for hedging derivatives. In practice, hedging a portfolio of derivatives is typically done by matching different sensitivities between the given portfolio and the hedging portfolio. As an alternative, a hedging portfolio can be chosen to minimise a measure of the hedge risk for a given time horizon.

The object of this paper is to analyze the optimality of some hedging strategies being offered to the insurer to cover the risks related to unit-linked life insurance contracts with minimum death benefit guarantee. These contracts are subjected to two types of risks: financial risk and mortality risk.

The financial risk is represented by the possibility of a poor evolution in the underlying asset, whereas the mortality risk results from the possibility of a strong fluctuation in the sample. In this last case, if the future mortality of the insured parties in the portfolio is stronger than foreseen, this may be due to the non-validity of the assumption of mutualisation of the deaths retained during the evaluation of the contract.

2. Risk-neutral valuation

Except for the reasons already noted in the Introduction, another reason for using risk-neutral valuation comes from the microeconomic theory of the uncertain. Indeed, let us remind ourselves of two of the founder assumptions:

- Individuals strictly prefer more income to less income; (or the equivalent less loss rather than more loss);
- Individuals are risk adverse.

The consequences of these assumptions on the agent's choices are strong. Indeed, a risk adverse individual prefers to have the expectation of the random variable with probability 1 rather than having a random variable where the probability is unknown. This means that between two games with identical expectations of earnings, the agent will choose the least risky. However, they will be inclined to change their choice if an additional amount is proposed to them. This amount is the risk premium.

The fair value must integrate this risk premium as this is what reflects the risk adverse character of investors on the markets. The incorporation of this risk premium allows the passage from the initial environment to a risk-neutral environment. The valuation of financial assets is generally made in this risk-neutral framework. The passage to this universe is made through the formulae of a change in probability, as justified by the Theorem of Radon–Nikodym.

The theory of deflators is an alternative to the risk-neutral valuation. Generally used in Assets–Liabilities Management (ALM), the deflators are stochastic factors of actualization which make it possible to predict the future flows of the liability. They allow us to obtain a "Market Consistent" valuation of projected flows, i.e. to find the initial value of risky assets.

Essentially, the use of deflators and the risk-neutral valuation are equivalent. Indeed, the deflator is nothing other than the density of the risk-neutral measure according to the historic measure. The existence of this density results from the Radon–Nikodym Theorem.

The neutral density risk (or the deflator) depends on the nature of the studied risk. Within the framework of our study we shall be confronted with two risks as mentioned above: mortality risk and financial risk.

To begin with, we shall accept the collectively accepted assumption about the risk of mortality, namely: "the perfect mutualisation of the deaths".[2] Accordingly, the mortality risk "disappears", in the sense that we can foresee with certainty the future number of deaths. Having said this, a mortality risk premium can be introduced by modelling mortality prudentially. In this case, the question of the level of prudence to adopt is open.

For the financial risk involved in managing the contracts, we will restrict ourselves to the Markovian models and apply them to an efficient environment. Thus, we make the assumption of an absence of arbitrage opportunity. Within this framework, one of the standard results of the financial theory is that all the assets are martingales under the risk-neutral probability.

This specific character of assets under the risk-neutral probability, besides simplifying the calculations, allows us to resolve the problem of actualizing generated future flows. To have fair value it will be enough to generate the asset under the risk-neutral probability and to actualize using the free-risk rate.

3. Insurance portfolio

We assume that a portfolio is constituted by N policyholders who invest in a single financial asset. Policyholder i aged x_i invests in a single risky asset $(S_t)_{t\geq 0}$. The insurer gives a guarantee of K_i in case the insured i dies before retirement. In the case the policyholder i dies at time T the insurer will pay $S_T + [K_i - S_T]^+$ to the beneficiaries of the insured i. Note that $[K_i - S_T]^+$ is the pay-off of a European put option with maturity T and strike price K_i on the underlying asset $(S_t)_{0\leq t\leq T}$.

The engagement of the insurer with respect to the policyholder i is written: $e^{-rT_{x_i}} \left[K_i - S_{T_{x_i}}\right]^+ 1_{T_{x_i}\leq\tau_i}$. Where T_{x_i} is the time to death of the policyholder, τ_i is the maturity of the contract and r is the risk-free interest rate.[3]

[1] This is true even if the application of the strategy is not always desirable or even feasible in practice.

[2] We will reconsider this assumption in a later study.

[3] We assume this to be constant.