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An investigation of the rank transformation in multiple regression

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Abstract

Real world data often fail to meet the underlying assumptions of normal statistical theory. The rank transformation (RT) procedure is recommended and used in the context of multiple regression analysis when the assumption of normality is violated. There is no general supporting theory of the RT. In view of this, the current study examined the Type I error and power properties of the RT in terms of multiple regression. The investigation included both additive and nonadditive models. Results indicated that there were severely inflated Type I error rates associated with the RT procedure under both normal and nonnormal distributions (e.g., 0.772 with nominal alpha = 0.05). The RT also exhibited a substantial power loss relative to the usual ordinary least squares regression procedure. It is recommended that the RT be avoided in the context of multiple regression despite its encouragement from SAS and other well respected sources.

Keywords: Type I error; Power; Rank transformation; Monotonic regression

1. Introduction

Micceri (1989) collected over 400 large real-world data sets and tested the assumption of normality for each distribution. Using the Kolmogorov–Smirnov test and a significance level (α) of 0.01, Micceri (1989) found all the distributions to be significantly nonnormal. It is well known that when the assumption of normality is violated, nonparametric tests can be substantially more powerful than the usual

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parametric t or F tests. For example, the Mann–Whitney test, when juxtaposed to the two independent samples t test, has an impressive asymptotic relative efficiency of 3 when the populations have an exponential distribution (see, e.g., Conover, 1999).

With respect to regression analysis, an additional concern to the assumption of a normally distributed error term is the assumption of a linear regression function. In view of these concerns, Iman and Conover (1979) introduced a nonparametric regression procedure that only requires the assumption of a monotonic regression function (i.e., linear or nonlinear). This procedure conducts the usual ordinary least squares (OLS) regression analysis on the ranks of the original scores. Thus, what makes the rank transformation (RT) in regression appealing is its simplicity and ease of execution. Specifically, the steps for hypotheses testing are (Iman and Conover, 1979): (1) separately replace the original scores of the dependent and k independent variables with their respective rank order, (2) apply the regular OLS regression procedure to the ranks, and (3) refer to the usual table(s) of percentage points to test the model.

Iman and Conover (1979) found favorable results for simple and multiple RT regression analyses using two nonnormal data sets from Daniel and Wood (1971). Recently, Conover (1999) submitted that the RT multiple regression procedure results in “a robust regression method that is not sensitive to outliers or nonnormal distributions to the extent that the regular regression methods on the data are affected” (p. 420).

Further, manufacturers of statistical software also promote the application of the RT in multiple regression. For example, the current SAS (1999) procedures guide states: “You can investigate regression relationships by using rank transformations with a method described by Iman and Conover (1979)” (p. 840). The IMSL (1994) manual also states, “Many of the tests described in this chapter may be computed using the routines described in other chapters after first substituting ranks for the observed values” (p. 582).

Recent suggestions promoting the use of the RT in other complex designs have also been made (Choi, 1998; Regeth and Stine, 1998). For example, Regeth and Stine (1998) submitted, “for two-way designs (involving an interaction), the ANOVA test can be run, using the rank orderings of data points rather than the actual scores” (p. 708). O’Gorman and Woolson (1993) also reported that the RT performed favorably in the contexts of logistic regression and discriminant analysis.

It is also important to point out that the RT has recently been used in applied studies involving complex designs. Some examples include multiple regression (Angermeier and Winston, 1998) and factorial ANOVA (Augner et al., 1998). It should be noted that the aforementioned suggestions and applications of the rank transformation to the general linear model have been made despite studies that have demonstrated numerous limitations of the RT in terms of simple regression (Lee and Yan, 1996) and other complex designs (e.g., Akritas, 1990; Brunner and Neuman, 1986; Sawilowsky et al., 1989; Thompson, 1991, 1993). For example, Lee and Yan (1996) demonstrated that the estimated RT regression slope coefficient lacks the asymptotic property of consistency except under trivial conditions.