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Incentive compatibility constraints and dynamic programming in continuous time

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Abstract

This paper is devoted to the study of infinite horizon continuous time optimal control problems with incentive compatibility constraints that arise in many economic problems, for instance in defining the second best Pareto optimum for the joint exploitation of a common resource, as in Benhabib and Radner [Benhabib, J., Radner, R., 1992. The joint exploitation of a productive asset: a game theoretic approach. Economic Theory, 2: 155–190]. An incentive compatibility constraint is a constraint on the continuation of the payoff function at every time. We prove that the dynamic programming principle holds, the value function is a viscosity solution of the associated Hamilton–Jacobi–Bellman (HJB) equation, and that it is the minimal supersolution satisfying certain boundary conditions. When the incentive compatibility constraint only depends on the present value of the state variable, we prove existence of optimal strategies, and we show that the problem is equivalent to a state constraints problem in an endogenous state region which depends on the data of the problem. Some economic examples are analyzed. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

In this paper, we study optimal control problems in continuous time/infinite horizon with *incentive compatibility constraints*. We consider a classical infinite horizon optimal control

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problem with a constraint on the continuation value of the plan at each time $t \geq 0$. The continuation value at each time t is constrained from below by a function of the state and of the control at time t. The constraint can be interpreted from an economic point of view as an outside option/incentive compatibility constraint.

This type of constraint gives rise to two different problems: optimal stopping problems and optimal control problems with incentive compatibility constraints. In the first case, we have a *positive* perspective: we want to study the optimal behavior of an economic agent in a dynamic setting allowing him/her at any time in the future to stop the process and to exercise an outside option which gives him/her a reward, which is a function of the state (termination payoff). In this setting, the set of admissible controls includes those violating the incentive compatibility constraint. The agent is allowed to get the termination payoff. In the second case, we have a *normative* perspective. The point of view is one of a social planner who wants to characterize optimal contracts or second best solutions to dynamic problems under incentive compatibility constraints for the agents of the economy. The set of admissible controls does not include those violating the incentive compatibility constraint. The social planner looks for an optimal policy among the policies, which do not include the termination of the process. The goal is the definition of a social contract taking into account the fact that the agents can decide in the future to go out of the contract. Such an event is prevented by including the incentive compatibility constraint. This type of problems is analyzed in this paper.

Many economic problems can be put in this setting. They belong to two main classes of models of the so-called second best literature. The first type of problems comes from the study of differential games such as the exploitation of an exhaustible common resource. The second type of problems arises in analyzing policy making without full precommitment (time consistency problems). In Section 2, we will fully describe a second best problem arising in the analysis of a differential game for the exploitation of a common exhaustible resource.

It is well recognized that in differential games such as the exploitation of a common resource. (Benhabib and Radner, 1992; Dockner and Sorger, 1996; Tornell and Velasco, 1992; Dutta and Sundaram, 1993a,b), capital accumulation (Fudenberg and Tirole, 1983), pollution, voluntary provision of a public good (Dockner et al., 1996), the outcome of the noncooperative interaction obtained as a subgame perfect equilibrium or as trigger strategy equilibrium may be Pareto inefficient, i.e. the reward for the agent is smaller than the one obtained by a representative agent under perfect competition (the so-called *tragedy of commons*). This result leads to the problem of designing contracts which are efficient among the subgame perfect equilibria (see Rustichini, 1992; Benhabib and Rustichini, 1996) and to the problem of designing contracts yielding at every time in the future a utility level higher than the one obtained according to a specific subgame perfect equilibrium. This type of problems can be formalized as the maximization of the utility of the representative agent under the constraint that at every time in the future, the continuation value of the consumption plan is greater than the utility obtained from the strategy of a subgame perfect equilibrium. Typically, the constraint is given by the value function of a control problem without constraints. The second class of models comes from optimal taxation problems in an intertemporal setting without full precommitment or in great generality from the analysis of an economy where there is a private sector and the government. The problem is the