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Factorization of second-order elliptic boundary value problems by dynamic programming

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Abstract

We present a method to factorize a second-order boundary value problem in a cylindrical domain in a system of uncoupled first-order initial value problems, together with a nonlinear Riccati-type equation for functional operators. This uncoupling is obtained by a space invariant embedding technique along the axis of the cylinder. This method can be viewed as an infinite-dimensional generalization of the block Gauss LU factorization.
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1. Introduction

In [1] Angel and Bellman proposed a method based on invariant embedding to transform a second-order elliptic boundary value problem in a rectangle in a system of first-order decoupled initial value problems which can be solved by a two sweep process (see also [2]). This formulation was derived only formally with the use of the Neumann-to-Dirichlet (NtD) map. Here we study this method for a model problem: the Poisson equation. In

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Section 2 we present the formal derivation of the factorization, extending previous results to an n-dimensional cylindrical domain with axis parallel to the x_1 coordinate, for various boundary conditions, using also the Dirichlet-to-Neumann (DtN) map. The first objective of the paper, carried out in Section 3, is to give a functional space framework and a mathematical justification of the derivation of the factorization. Secondly, in Section 4 we show the relation of the use of invariant embedding for this problem and for optimal control problems associated to evolution equations. In particular, we show the relation between the time dependent Riccati equation providing the feedback law of such optimal control problems (see e.g. [9,5]) and the x_1 Riccati equation satisfied by the NtD or DtN maps in our case. We also show, in Section 5, that this factorization can be viewed as the extension to the infinite-dimensional problem of the well-known block Gauss LU factorization of the matrix of the discretized problem. Section 6 gives some clues about the interest of such a factorization for the study of elliptic boundary value problems, presenting some situations where one can take advantage from the factorized form of the problem. It is believed that the method of factorization of boundary value problems is more general and can be applied to more complex situation than the Poisson equation in a cylindrical domain. We found this case convenient to present the method and give full mathematical justifications. Other results can be found in [15]. In [7], the authors use these techniques (in a formal way) to solve an optimal control problem associated to an elliptic equation and get the optimal control in an explicit way. In [6] the method is applied to the factorization of the linear elasticity system. Furthermore, similar techniques have been used recently in acoustics in order to compute generalized impedance in waveguides (see [13,12]). Specific numerical schemes are developed from this approach [11].

2. Elliptic problem in a cylindrical domain

We consider the Poisson equation in a cylindrical domain along the x_1-coordinate. This coordinate plays the role of time for a parabolic equation. We shall make a strong analogy with the uncoupling of the optimality conditions associated to an optimal control problem of such systems.

2.1. Statement of the problem and formal resolution

Let $\mathcal{O}$ be a smooth bounded open set in $\mathbb{R}^{n-1}$, Ω be the cylinder $\Omega =]0,a[\times\mathcal{O}$ in $\mathbb{R}^n$, $\Gamma_0 = \{0\}\times\mathcal{O}$, $\Gamma_a = \{a\}\times\mathcal{O}$ and $f \in L^2(\Omega)$. The lateral boundary of the cylinder is denoted by $\Sigma = \partial\mathcal{O}\times]0,a[$. The regularity of the data y_0 and y_1 is defined below. Let us denote $\Delta = \sum_{i=1}^{n}\frac{\partial^2}{\partial x_i^2} = \frac{\partial^2}{\partial x_1^2} + \Delta_z$, where z denotes the independent variables $x_2,\dots,x_n$. We consider the problem

$$(\mathcal{P}_0) \quad \begin{cases} -\Delta y = f & \text{in } \Omega,\\ y|_\Sigma = 0, \quad -\left.\dfrac{\partial y}{\partial x_1}\right|_{\Gamma_0} = y_0, & y|_{\Gamma_a} = y_1. \end{cases}$$

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