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Possibility distributions of fuzzy decision variables obtained from possibilistic linear programming problems

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Abstract

In this paper, several kinds of possibility distributions of fuzzy variables are studied in possibilistic linear programming problems to reflect the inherent fuzziness in fuzzy decision problems. Interval and triangular possibility distributions are used to express the non-interactive cases between the fuzzy decision variables, and exponential possibility distributions are used to represent the interrelated cases. Possibilistic linear programming problems based on exponential possibility distributions become non-linear optimization problems. In order to solve optimization problems easily, algorithms for obtaining center vectors and distribution matrices in sequence are proposed. By the proposed algorithms, the possibility distribution of fuzzy decision variables can be obtained. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

For some considerable time, linear programming (LP) has been one of operational research techniques, which has been widely used and got many achievements in both applications and theories. But the strict requirement of LP is that the data must be well defined and precise, which is often impossible in real decision problems. The traditional way to evaluate any imprecision in the parameters of an LP model is through a post-optimization analysis, with the help of sensitivity analysis and parametric programming. However, none of these methods is suitable for an overall analysis of the effects of imprecision in parameters. Another way to handle imprecision is to model it in stochastic programming problems according to the probability theory. A third way to cope with imprecision is to resort to the theory of fuzzy sets, which gives the conceptual and theoretical framework for dealing with complexity, imprecision and vagueness [4,5,13].

Generally speaking, in fuzzy linear programming problems, the coefficients of decision variables are fuzzy numbers while decision variables are crisp ones. This means that in an uncertain environment, a crisp decision is made to meet some decision criteria. On the other hand, Tanaka et al. [6] initially proposed a possibilistic linear programming formulation where the coefficients of decision variables are crisp while decision variables are obtained as fuzzy

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numbers, and LP technique is used to obtain the largest possibility distribution of the decision variables. As an extension of that idea, Guo et al. [3] have used LP and quadratic programming (QP) techniques to obtain different fuzzy solutions to enable a decision maker select one preferable fuzzy decision among the obtained ones. Tanaka et al. [7] dealt with the interactive case in which exponential distribution functions are used.

Possibility distributions, which are the basis of the possibility theory, initially proposed by Zadeh [12] and advanced by Dubois and Prade [2] take the same roles as the probability distributions in probability theory. Generally speaking, the possibility distributions can be categorized into two main classes. One is the non-interactive case where the joint distribution of fuzzy variables is defined by the minimum operator and the possibility distribution of each fuzzy variable can be individually characterized by a well-known interval or a L–R membership function and so on. The other is the interactive case where the joint possibility distribution of fuzzy variables should be characterized by some distribution matrices, such as quadratic and exponential possibility distributions [9–11]. In this case, a possibility distribution with respect to the one-dimensional space can just be obtained as a marginal possibility distribution defined by the extension principle [12]. It is straightforward that different kinds of possibility distributions can depict different kinds of fuzziness in real decision-making problems where the type of the possibility distribution should be chosen by experts because fuzzy models are from the experience of experts. Based on the different possibility distributions, the possibilistic data analysis for decision making has been intensively researched by Tanaka and Guo [8].

This paper studies the possibility distributions of fuzzy decision variables in possibilistic linear programming problems to reflect the inherent fuzziness in fuzzy decision problems. From simple to complex, three kinds of typical possibility distributions, namely, interval, triangular and exponential possibility distributions are used to represent the non-interactive and interrelated cases between the fuzzy decision variables, respectively. Possibilistic LP problems can be directly reduced into conventional LP problems in interval and triangular cases. Since exponential possibility distributions are considered, possibilistic LP problems become non-linear optimization problems. In order to solve these optimization problems easily, algorithms for finding out center vectors and distribution matrices of exponential possibility distributions in sequence are proposed. By these proposed algorithms, fuzzy solutions can be obtained to reflect the inherent fuzziness in fuzzy decision problems with different kinds of possibility distributions.

2. Fuzzy decisions in possibilistic linear programming problems

Decision problems are often described in terms of a multilevel functional hierarchy. From the bottom to the top, the decision objectives become large and complex. For a lower level of decision-making systems, decision makers need to make a crisp decision under a fuzzy environment as widely studied by conventional fuzzy programming problems. By contrast, fuzzy decisions are often required in an upper decision level because giving some room for a lower decision level is necessary in real decision problems. The fuzzy decisions obtained in the upper level will be sent to the lower level where crisp decisions will be made within these fuzzy decisions. Hence, obtaining possibility distributions of fuzzy decision variables to reflect the fuzziness of decision problems is the aim of this paper.

Mathematically, a fuzzy decision problem can be described as

$$\begin{aligned} & b_{11}X_1 + \cdots + b_{1j}X_j + \cdots + b_{1n}X_n \geqslant C_1, \\ & \vdots \\ & b_{i1}X_1 + \cdots + b_{ij}X_j + \cdots + b_{in}X_n \leqslant C_i, \\ & \vdots \\ & b_{m1}X_1 + \cdots + b_{mj}X_j + \cdots + b_{mn}X_n \geqslant C_m, \end{aligned} \tag{1}$$

where b_{ij} is a crisp coefficient, C_i is a fuzzy number $(i=1,\dots,m)$ and the fuzzy decision vector $\boldsymbol{X}=[X_1,\dots,X_n]^t$ is governed by a possibility distribution. Owing to reflecting the inherent fuzziness in decision problems, the type of possibility distributions should be chosen by experts. In the following sections, three kinds of typical possibility distributions are investigated consecutively to reflect the non-interactive and interrelated cases between decision variables.