Pergamon

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International Journal of Machine Tools & Manufacture 43 (2003) 1023–1033

INTERNATIONAL JOURNAL OF MACHINE TOOLS & MANUFACTURE
DESIGN, RESEARCH AND APPLICATION

Linear-programming-based assessments of geometrical accuracy: standard presentation and application area

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Received 4 March 2002; received in revised form 7 March 2003; accepted 14 March 2003

Abstract

An application of the linear programming (LP) approach to form and position accuracy assessments based on the minimum zone (MinZ) method is considered. The standard form of the optimization function and constraints using linearized form of the substitute feature is considered. A linearization-caused error is estimated by reference to the second order terms within the Taylor series expansion of the rotation matrix. The acceptable range of the LP-based assessments is defined for some levels of an allowable error. The LP-based assessment for a surface is simulated. Actual measurements show that the LP-based estimations have equal or fewer values in comparison with those obtained by other methods.

Keywords: Geometric accuracy; 3D measurement; Accuracy assessment; Linear programming; Minimum zone

1. Introduction

A modern 3D metrology employs measurements on coordinate measuring machines (CMM) for observation of actual surfaces [1–3]. When one uses these measurements to estimate the accuracy of form and position, the calculation process contains an optimization procedure as a key step of the process. Depending on the type of optimization criterion, two main groups of assessments are applied: least mean square (LMS), and minimax. Until recently, only the LMS-type assessments were applied, but nowadays international and national standards for form and position tolerancing widely use minimum zone (MinZ) for accuracy assessments [2,3]. As an example, a comparison of two types of assessments for flatness is given in Fig. 1. The LMS-based assessment Δ_{LMS} is obtained as a sum $\Delta_{LMS} = \Delta_{max} + \Delta_{min}$ of two extreme point deviations from the mean plane (Fig. 1b), while the MinZ-based assessments are calculated as a distance Δ_{MinZ} between the MinZ boundaries (Fig. 1a). Among the typical minimax assessments, such as maximum inscribed, minimum circumscribed, and MinZ, we will consider only the last-listed assessment. When an accuracy of the surface or line is measured, the MinZ is built as follows: a substituted feature (i.e., the feature of the nominal form) is extracted from the measured points; the zone boundaries are built so that all the points of the actual surface lie between or on the boundaries of the zone, and the width of the zone is minimal.

It has been known that a procedure of the MinZ construction reduces to a solution of the minimax optimization problem [4–8]. Various approaches are known to solve this problem: statistical approach, computational geometry techniques, non-linear programming, linear programming, etc.

The Monte Carlo search [9] is a statistical method based on the random selection of variables defining the surface. Because of this random selection, this method requires many sampling points to assure high accuracy. Another statistical approach is described by Yang et al.

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0890-6955/03/$ - see front matter © 2003 Elsevier Science Ltd. All rights reserved.
doi:10.1016/S0890-6955(03)00092-0

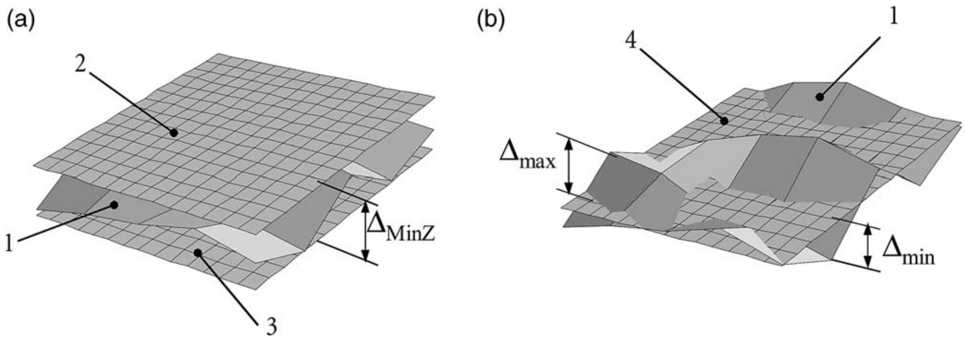

Fig. 1. (a) MinZ-based—and (b) LMS assessments for flatness: 1—actual plane; 2, 3—external and internal boundaries of the MinZ; 4—LMS plane.

[10]. Yang et al. used large sets of uniform sample points measured from five machined surfaces and compared the form error using individual points and fitted surfaces obtained through a spatial statistic method. This method takes into account a correlation between spatial coordinates of the sample points. A large number of samples (on the order of 1000) is required to make this method feasible.

In the widespread computational geometry technique, specific geometry features are built, such as convex hulls, Voronoi diagrams, and so on. A convex hull algorithm [6,11–14] is based on constructing the minimum polygon that will enclose all the measurement points. For straightness and flatness, Traband et al. [11] presented a method based on the concept of a convex hull of the measurement data. The algorithm for evaluating flatness tolerance based on constructing a 3D convex hull is described by Lee [13]. The overall solution procedure consists of three stages, and the complexity of this method becomes $O(N^2 \log N)$, where N is the number of sampled points. As described by Lin et al. [12], the convex hull in 3D space is a minimum polyhedron that encloses all the given points. To compute this polyhedron, the projection of these points in the *XY*, *XZ*, and *YZ* planes must be first computed. These convex hulls are then merged together to obtain set of vertices for the polyhedron. The procedure for calculating MinZ is reduced to calculating the distance of each point from each of the faces of the convex polyhedron. Samuel et al. [14] developed an algorithm consisting of five steps for function-oriented evaluation of straightness and flatness using minimum and maximum enveloping features. The overall complexity of this algorithm is $O(N \log N)$.

The Voronoi diagrams method [9,15,17] is based on collecting the nearest or farthest 'neighborhood' for each of the measurement points. The circularity assessment method proposed by Murthy et al. [9] is based on the construction of both the nearest Voronoi diagram and the farthest Voronoi diagram and then finding the intersection of the two. Roy and Xu [16] proposed a computational technique for the cylindricity assessment which is based on measurements of actual cylindrical surfaces in some cross-sections. The 2D MinZ is built for each cross-section using Voronoi diagrams. The axis of the 3D MinZ is obtained by means of an LMS technique, and then the diameters of the tolerance zone are established as a minimum and maximum of the diameters in the cross-section. In this method, it is necessary to have more than three points in each cross-section; therefore, this method cannot be applied if, for example, the measurement points are located along the helical line. Huang [17] used 3D Voronoi diagrams to construct the MinZ for sphericity evaluation. The Voronoi diagram in a 3D space is defined as a bisector plane between two measured points, where any point on plane is of equal distance to each of the corresponding measured points.

Non-linear programming [5,8,18,19] is applied when the problem is formulated as a non-linear optimization problem with respect to the optimization parameters. It must be noted that non-linear algorithms are rather complicated. Wang [5] proposed a non-linear optimization method for the straightness and flatness MinZ evaluation. The computational results show the relatively high performance of the optimization method; however, convergence to local optimal solutions due to the non-convex objective function and computational complexity of the problems will be a critical limitation on its practical use. Radhakrishnan et al. [7] described the application of an iterative cyclic coordinate algorithm to minimax non-linear cylindricity estimation problem. The complexity of this algorithm is shown to be $O(N^6)$, where N is the number of sampled points. Choi et al. [8] formulated the construction of the MinZ as a non-linear unconstrained optimization problem and used an iterative search technique to solve it. Orady et al. [19] developed an iterative method for the MinZ evaluation of the cylindricity. The proposed algorithm consists of seven steps, beginning with the fitting by LMS.

The LP procedure used in the following papers has well-established advantages over other minimax optimization methods [20], such as a rich variety of effective algorithms and software, fast computing, flexibility with