Interior point algorithm for linear programming used in transmission network synthesis

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Abstract

This article presents a well-known interior point method (IPM) used to solve problems of linear programming that appear as sub-problems in the solution of the long-term transmission network expansion planning problem. The linear programming problem appears when the transportation model is used, and when there is the intention to solve the planning problem using a constructive heuristic algorithm (CHA), or a branch-and-bound algorithm. This paper shows the application of the IPM in a CHA. A good performance of the IPM was obtained, and then it can be used as tool inside algorithms used to solve the planning problem. Illustrative tests are shown, using electrical systems known in the specialized literature.

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1. Introduction

The expansion of electrical transmission systems should be based on an optimal plan, i.e., the plan should specify the transmission lines and/or transformers needed for the system to operate efficiently in relation to a given planning horizon. The parameters requiring consideration include the topology of the base year, candidate circuits, data about generation and demand for the planning horizon, and investment constraints. Thus, the solution for a planning problem should specify where, how much and when new equipment for expansion should be installed. There are two types of planning: static planning involves a single planning horizon, but multi-stage planning is a derived generalization considering the separation of the planning horizon into various stages.

The expansion of transmission systems is generally modeled mathematically using the so-called DC model, which involves mixed non-linear programming, but its application is problematic for large-scale systems. Various modifications have thus been introduced, including relaxed versions of the DC power flow model. Greater details about the mathematical modeling of transmission system planning can be found in [1].

Many algorithms for the solution of problems involving the planning of transmission systems have been proposed in the specialized literature. These can be separated into three categories: (a) heuristic algorithms, (b) classic optimization algorithms such as Benders decomposition and branch and bound algorithms, and (c) meta-heuristic such as simulated annealing (SA), genetic algorithms (GA), and tabu search (TS).

The focus of this research is the use of IPM algorithms in order to solve problems of linear and non-linear programming that appear as part of the solution process in algorithms used to solve the problem of long term transmission network expansion planning, also known as network transmission synthesis. Problems of linear programming (LP) or non-linear programming (NLP) should be usually solved in the implementation of the three categories algorithms used in the...
planning of transmission systems, mentioned earlier. In fact, the longer processing time spent by these algorithms happens while solving the problem of LP or NLP. The shape of these problems varies slightly according to the kind of optimization algorithm used to solve the planning problem.

This article presents an IPM to solve a LP problem that appears when the constructive heuristic algorithm of Garver is used in the transportation model. The presented algorithm may be easily adapted to solve problems of LP that appear when there is the intention to solve the transportation model using an algorithm of branch-and-bound.

2. The transportation model

The Transportation Model (TM) for static planning was originally proposed by Garver as a simplification of the standard DC model [2]. It involves the relaxing of the non-linear constraints in the DC model, thus giving rise to a more easily manipulated linear model. The resultant problem is a mixed integer linear programming (MILP) problem, which can be stated, in the following form:

\[
\begin{align*}
\text{Min} & \quad v = \sum_{(i,j) \in \Omega} c_{ij} n_{ij} \\
\text{s.t.} & \quad S f + g = d \\
& \quad |f_{ij}| \leq (n_{ij}^o + n_{ij}^f) \mathbf{t}_{ij} \\
& \quad 0 \leq g \leq \mathbf{\Gamma} \\
& \quad 0 \leq n_{ij} \leq \mathbf{\Pi}_{ij} \\
& \quad n_{ij}, \text{ integer}; \quad f_{ij}, \text{ unbounded} \\
\end{align*}
\]

where \(c_{ij}, \gamma_{ij}, n_{ij}, n_{ij}^o, n_{ij}^f, \mathbf{t}_{ij} \) and \(\mathbf{\Pi}_{ij} \) represent, respectively, the cost of a circuit which is a candidate for addition to the right-of-way \(i \rightarrow j\), the susceptance of that circuit, the number of circuits added to the right-of-way \(i \rightarrow j\), the number of circuits in the original base system, the power flow in the right-of-way \(i \rightarrow j\), and the maximum power flow per circuit in the right-of-way \(i \rightarrow j\). The value \(\mathbf{\Gamma} \) is the investment, \(S\) the transpose branch-node incidence matrix, \(f\) a vector with elements \(f_{ij}\), \(g\) a vector with elements \(g_{ij}\) (generation in bus \(k\)) with a maximum value of \(\mathbf{\Pi}_{ij}\), the maximum number of the circuits that can be added in the right-of-way \(i \rightarrow j\), and \(\Omega\) is the set with indices of all circuits.

Constraint (2) in the TM represents power conservation in each of the nodes as a model of Kirchhoff’s Current Law (KCL) in the equivalent DC network.

The transportation model has the advantage of being linear and therefore, it is easier to solve than the DC model, which is non-linear. Both problems are, however, mixed integers. It is possible to find the optimal solution for the transportation model using algorithms of the kind of branch-and-bound as presented in [3]. Nevertheless, for large and complex electrical systems the branch-and-bound algorithms have prohibitive processing times. A disadvantage of the transportation model is that the optimal solution of this model is usually unfeasible for the DC model that is considered ideal for applications in transmission network planning. Therefore, more important than finding the optimal solution for the transportation model is finding a topology or solution that presents less infeasibility for the DC model.

The transportation model is an MILP and any optimization technique can be used in order to find the optimal solution for this kind of problem—Benders, branch-and-bound, etc.

It was Garver who presented for the first time the transportation model and a constructive heuristic algorithm to solve this kind of model [2]. A constructive heuristic algorithm (CHA) is a process of iterative solution to find a good solution for a complex problem, through a step-by-step process. A component of the solution is added at each step and the process ends when a feasible high quality solution, is found. A CHA is robust and converges quickly, but for large and complex problems this kind of algorithm converges in a good quality solution only, which may sometimes be very far from the optimal solution.

At each step of a CHA, a component of the solution to the problem—a circuit must be added to the system in the case of the planning problem—must be chosen. The choice of this component (circuit) is determined by a sensitivity index of the CHA. Garver’s main idea is in the kind of sensitivity index proposed. Garver suggests to solve, at each step of the algorithm, the transportation model itself, after relaxing the integrality of the investment variables. In other words, if we make \(n_{ij} \geq 0\) in the TM, the TM will be transformed into a simple linear programming (LP) problem. The solution of this LP may imply the most important circuit that must be added to the system in the current stage of implementation of the CHA. Garver suggests that the circuit with \(n_{ij} = 0\) must be added to the system, and that it should take the larger power flow in the solution of the LP, that is, the circuit with greater value of \(f_{ij} = n_{ij} \mathbf{t}_{ij}\). The Garver algorithm, after a few changes in [2] takes the following form:

1. Undertake the base topology as the current topology with known values of \(n_{ij}^o\).
2. Solve a LP problem for the relaxed transportation model \((n_{ij} \geq 0)\) and for the current topology. If the LP problem solution indicates \(v = 0 \Rightarrow n_{ij} = 0\) stop, because a feasible solution for the TM was found, and go to step 4. Otherwise, go to step 3.
3. Sensitivity index in the LP solution, identify the new circuit that takes the larger power flow. Update the current topology \((n_{ij}^f)\) adding the circuit identified by the sensitivity index. Go to step 2.
4. Order the added circuits in decreasing order of the costs. Using a LP, at each step, check that the removal of a circuit keeps the system in adequate operational condition.
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