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# A linear programming approach for identical parallel machine scheduling with job splitting and sequence-dependent setup times

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**Abstract**

In this study, we consider the problem of scheduling a set of independent jobs with sequence-dependent setup times and job splitting, on a set of identical parallel machines such that maximum completion time (makespan) is minimized. For this NP-hard problem, we suggest a heuristic algorithm improving an existing one, using a linear programming modeling with setup times and job splitting considerations. The performance of our new method is tested on over 6000 instances with different size by comparing it with a lower bound.


*Keywords:* Scheduling; Parallel machine; Splitting; Setup times; Linear programming

## 1. Introduction

This paper considers an identical parallel machine scheduling problem (PMSP) in which setups are necessary between the processing of different jobs. The criterion is to minimize the makespan. This problem is NP-hard (Nait et al., 2003; Yalaoui and Chu, 2003; Sveltana et al., 2001). The problem of scheduling jobs on identical parallel machines, without preemption, to minimize makespan is NP-hard, since even the problem restricted to two machines ($P2//C_{\max}$) was shown to be NP-hard by a reduction to a bi-partitioning problem by Karp (1972).

Note that our problem is a real-life one, as most of the studies on PMSP taking into account setup times in order to minimize makespan. The setup time has often been considered to be negligible or as a part of the processing time. Otherwise, we can distinguish two types of setup time: *sequence-independent* and *sequence-dependent* setup times. *Sequence-independent* setup time depends only on the job to be processed while sequence-dependent setup time depends on both the job to be processed and the one yet processed.

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The phenomenon of sequence-dependent setup times has been investigated by researchers for real-world job-shop environment as glass industry (Paul, 1979; Chevalier et al., 1996), metallurgical industry (Narasimhan and Panwalkar, 1984; Narasimhan and Mangiameli, 1987), paper industry (Sherali et al., 1990), chemical industry (Riane, 1998) and (Fortemps et al., 1996), textile industry (Sherali et al., 1990; Aghezzaf et al., 1995), wood industry (Riane, 1998), aerospace industry (Li, 1997).

Another important feature of our problem is that each job can be split into job sections, which can be processed in parallel on different machines. In textile industry for example, a job can be a batch of several parts (100 shirts or 1000 socks ...). Any batch can be decomposed into sub-batches. This corresponds to the splitting of jobs. The difference between job splitting and preemption is that job sections cannot be processed on different machines simultaneously if only preemption is allowed. However, with job splitting, jobs can be split to continuous sublots and processed simultaneously on different machines.

The first on parallel machines scheduling problem dates from the Fifties through the works of McNaughton (1959) and Hu (1961). This type of problem has received continuous interest from researchers and from this fact the literature increased. Various constraints have been taken into account and different criteria have been studied.

The first heuristic to solve PMSP based on the nearest neighbor was published in 1978 (Frederickson et al., 1978). Other authors investigated the problem, such as Flynn (1987), Hahn et al. (1989) and Bobrowski and Kim (1994, 1997).

They investigated the impact of setup time variation on sequencing decision. Guinet (1993) shows that PMSP can be reduced on vehicle routing problem (VRP) and suggest first a two step heuristic. Secondly, they formulated the problem in a zero–one linear program and proposed a new heuristic which consisted of solving the dual problem of the obtained assignment problem. A state-of-the-art until 1990 on parallel machines scheduling according to the viewpoints: completion-time-based, due-date-based and flow-time-based performance measures, is given in Cheng and Sin (1990). Lam and Xing (1997) gave a short review of new developments of PSMP associated with the problems of just-in-time productions, preemption with setup and capacited machine scheduling.

Xing and Zhang (2000) presented a heuristic for identical PMSP with sequence-independent setup times and job splitting which has the worst-case performance ratio not greater than $\frac{7}{4} - 1/M$ for $M \geqslant 2$ where $M$ is the number of machines. They also presented a linear programming formulation for an unrelated PMSP with job splitting and without setup times to minimize makespan. We note also that most of works using linear programming deal with unrelated parallel machines (Potts, 1985; Xing and Zhang, 2000). For more information, we refer interested readers to the papers of Allahverdi et al. (1999) and Yalaoui and Chu (2003).

The proposed new method is an improvement of the method proposed by Yalaoui and Chu (1999, 2003) and called Y_C. Heu thereafter. This latter method can be decomposed into two parts. Firstly, the PMSP with sequence-dependent setup times is reduced to a single machine scheduling problem (SMSP). The SMSP is transformed into a traveling salesman problem (TSP) and solved by an appropriate algorithm. Secondly, a feasible initial solution to the original parallel machine problem is obtained by using the result of the first part. This initial solution is then improved using an iterative algorithm in a step by step manner, taking into account the setup times and job splitting. The idea is to reduce makespan at each iteration by sublots calibration between critical machines. The computations stop when the improvement becomes insignificant. It means that the iteration number may not be bounded.

Many works have been performed on the subject. Riotteau et al. (2001) proposed an improvement of the lower bound of Yalaoui and Chu (1999), and a new heuristic based on a multiple TSP resolution. The TSP problem is solved to optimality using appropriate algorithms. Therefore the number of jobs is limited and important computational time is required.