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A stochastic linear programming approach for service parts optimization

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ABSTRACT

Service Parts Stock Management is a part of the service process to ensure that right spare parts are in the warehouse at the right place and time, with respect to a customer demand. Customer satisfaction can be measured through the First Fill Rate Value (FFRV). The FFRV optimization is an inventory planning problem, which can be formalized through a stochastic linear programming problem and evaluated for different spare parts or sets of them, considering production and cost constraints. In this paper the problem and the details of the proposed approach will be discussed and assessed through some experimental data.

1. Introduction

Service Parts Management is one of the main components of strategic service logistics, requiring a complex decision making process [1], that companies use to ensure that right spare parts and resources are at the right place at the right time. From a producer point of view spare parts are considered uneconomical since they involve logistical and economical requirements; they might never be used and the cost of inventory on hand is not negligible [2,3]. However, without spare parts on hand, a company's customer satisfaction levels could drop, since the customers has to wait for long time before their products can be fixed.

The company's trade-off is between spare parts inventory and resources to achieve optimal customer satisfaction levels with minimal costs. As suggested by Ref. [4], fill rate is a simple but effective measure of inventory availability and customer satisfaction can be related to the First Fill Rate Value (FFRV). In particular, FFRV is a measure of fulfilment quality, corresponding to the ratio between the order items satisfied by the available inventory and the total number of received order items. FFRV can be evaluated for different spare parts or sets of them [5], taking into account production and cost constraints.

In this paper we propose a solution to this inventory optimization by approaching it through the definition and the solution of a stochastic linear programming problem (SLPP). This planning involves quantities and variables associated with a sequence of time intervals (called *time buckets*) composing the planning horizon. In the proposed approach the quality constraints are based on a piecewise linear regression of FFRV identified by means of the analysis of historical data.

The focus of the work presented in this paper is to develop and experiment an optimization model that is appropriate for facing the inventory planning problem, i.e., that is able to suitably set the safety factors taking into account costs and levels of service. The problem will be solved with a scenario-generation method, firstly formally defining the considered Inventory Stock Mix Optimization Problem (ISMOP) and then introducing the details of the proposed approach. The effectiveness of procedure will be evaluated by presenting some experimental results.

2. The optimization problem

The ISMOP here considered consists in determining the optimal safety stock levels for the stock of a set of spare parts so that the total production and inventory cost is minimized, while a set of quality constraints are satisfied.

We consider an *optimization horizon T* consisting of a sequence of time intervals, the so-called *time buckets*, of a given fixed duration (e.g., a week or a month); then we consider the optimization decisions relevant to the time buckets $t = 1, \ldots, T$. For each bucket t we assume that a forecast f_{pt} for the stock that should be available for each spare part (or product) $p \in P$ can be computed from the analysis of historic data; in particular, such forecast includes two components:

$$f_{pt} = a_{pt} + s_{pt} \quad (1)$$

The a_{pt} component represents the average forecast, which takes into account both the past average demand and its regularity, whereas s_{pt} is the *safety stock level* for spare part p in the time bucket t, which is introduced to consider in the forecast the possibility of significant variations from the average. The higher is the safety stock level the greater is the capability of the considered supplier to face single and isolated peaks in the demand; on the other hand, if such a capability is generally spread for all the products in any time bucket, the consequent production and inventory costs cannot usually be accepted. The ISMOP hence corresponds to determine which is the optimal level of the safety stock for the products in each time bucket, minimizing the production and inventory cost, while taking into account the quality of service.

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The historical data consist of a collection of spare part orders received from customers in a past period that is long enough to produce reliable estimates, but not too dated to make such estimates sensitive to the demand trends in the recent past. As an example, we can assume to collect data for the most recent past time interval whose duration is twice the one of the optimization horizon T. In addition, as customer orders usually correspond to a set of spare parts' requests, we denote as *order items* the single requests included in a customer order; such an information will be used in the following to define a specific quality measure.

We assume that for each spare part p and each past time bucket $t = -2T, -2T + 1, \ldots, 0$, the following data were collected:

- C_p: handling cost for product p;
- Q_{pt}: total quantity of product p ordered in the bucket t;
- O_{pt}: the number of order items for product p in time bucket t;
- D_{ptr}: the quantity of product p ordered in order item r in time bucked t.

On the basis of such sets of data we estimate, by using a forecast model (derived from a time series analysis), for each product p and each future time bucket in the optimization horizon t, the following quantities:

- μ_{pt}: the average forecast;
- p_{pt}: the standard deviation.

Note that the reliability of μ_{pt} and σ_{pt} depends on the cardinality of the sample data set for past time buckets and products. Then, the forecast (1), indicating the stock of a spare part p that must be available in order to satisfy the demand in a bucket t can be modelled as

$$f_{pt} = \lambda_p \mu_{pt} + k_p \sigma_{pt} \tag{2}$$

where $a_{pt} = \lambda_p \mu_{pt}$ and $s_{pt} = k_p \sigma_{pt}$.

The quantity λ_p is the *frequency factor*. It is calculated by analyzing historical data occurrences in order to take into account the regularity of demand; differently from σ_p, λ_p does not depend on how the magnitude of the samples is spread, but on the regularity of the occurrence of spare part demand over time. As an example, if a product p was ordered in all the past time buckets considered, then we obtain $\lambda_p = 1$; on the other hand, if in the historical data we found an order item in a single past bucket, even if for a large quantity, then λ_p will be close to zero.

The quantity k_p is called *safety factor* and is used as a standard deviation multiplier for tuning the safety level as a function of the product; the safety factor must be fixed so that

$$0 \le k_p \le k_p^{\max}$$

being $k_p^{\max}$ the value required to avoid out-of-stocks having a probability π_p selected for each product p (e.g., for $\pi_p = 34\%$ we must fix $k_p^{\max} = 1$; for $\pi_p = 5\%$, $k_p^{\max} = 2$; for $\pi_p = 0.03\%$, $k_p^{\max} = 3$).

Solving the ISMOP corresponds to determine the values of the safety factors $k_p \ \forall p \in P$, which are the decision variables for the problem. The condition on the quality solution for the ISMOP is imposed by means of a lower bound for the *level of service* provided to customers. In particular, the set of products P is partitioned into a set of G clusters, i.e., $P = \bigcup_{g \in G} P_g$, $P_h \cap P_g = \emptyset$ $\forall h \ne g$, so that each cluster P_g includes products whose orders must be served with a same level of service. Then, a lower bound for the level of service, $\mathrm{LoS}_g \ \forall g \in G$, is specified as the minimum average service level that must be guaranteed for the products in P_g. The measure of the service level that we considered in this work is the FFRV (in the following FFR). FFR can be evaluated for past demand over a time interval, modelling the customer satisfaction as the ratio between the number of order items that were dispatched and the total number of order items included in the customer orders. We can express the minimum acceptability conditions on the level of service for each past time bucket t and each product cluster g as

$$\mathrm{FFR}_{gt} \ge \mathrm{LoS}_g \quad \forall g \in G, \ t = -2T, \ldots, 0 \tag{3}$$

where we compute the FFR_{gt} as

$$\mathrm{FFR}_{gt} = \frac{1}{|P_g|} \sum_{p \in P_g} \mathrm{FFR}_{pt} \quad \forall g \in G, \ t = -2T, \ldots, 0 \tag{4}$$

being FFR_{pt} the FFR for product p in the bucket t obtained as

$$\mathrm{FFR}_{pt} = \frac{E_{pt}}{O_{pt}} \quad \forall p \in P, \ t = -2T, \ldots, 0 \tag{5}$$

Note that in (5) E_{pt} and O_{pt} are respectively the number of order items dispatched and the total number of ordered items for product p in any of the past time bucket t. E_{pt} has been estimated through a forecast of historical data. The optimization objective of the ISMOP corresponds to minimize the total stock cost (TSC) for the forecasted plan. In particular, C_p is given for each $p \in P$ and TSC is computed for each time bucket t in the optimization horizon on the basis of the forecasts f_{pt} yielded by the forecast model as

$$\mathrm{TSC}_t = \sum_{p \in P} C_p f_{pt}, \quad t = 1, \ldots, T \tag{6}$$

The optimal safety factors k_p^* can be obtained from the minimization of (6) provided that a suitable computation model is used for accounting the quality service conditions (3) in forecasting, i.e., for the time buckets included in the optimization horizon.

3. The approach

The approach here proposed for the ISMOP is based on the definition and solution of a stochastic linear programming problem. Stochastic linear programming is an optimization approach used to solve linear problems when the assumption that all the model parameters are known for a certainty does not hold true [6]. Differently from the deterministic linear programming problem, in a SLPP some of the involved parameters are not constants but randomly generated, so that the objective function and/or some of the constraints are not deterministically computed or verified. In the following we will illustrate how we model the ISMOP as a SLPP.

The objective of the SLPP in this case is the minimization of the total stocking cost needed to satisfy the forecasted demand, so that the quality of service is guaranteed by imposing a minimum service level for the clusters of products. The decisions then correspond to determine the product safety factors k_p for $p \in P$ in each future time bucket $t = 1, \ldots, T$, that is, to determine the optimal stock quantity f_{pt} for the products in those buckets. In order to model the conditions on the quality of service we need to identify a relationship between the stock quantities f_{pt} and the corresponding FFR_{pt}. This latter index, computed for historical data as in (5), clearly depends on the ratio A_{pt}/Q_{pt} between the quantity of product p actually available in a bucket t, A_{pt}, and the total quantity of product ordered in t, Q_{pt}. If $A_{pt}/Q_{pt} \ge 1$ then any received order item can be dispatched giving $\mathrm{FFR}_{pt} = 1$, whereas if $A_{pt}/Q_{pt} < 1$ the FFR_{pt} decreases to zero when A_{pt} decreases to zero. In any time bucket t in the optimization horizon we consider an estimation of the FFR, $\widehat{\mathrm{FFR}}_{pt}(A_{pt}/Q_{pt})$, setting A_{pt} equal to the stock quantity to be identified f_{pt}, and modelling the total order quantity Q_{pt} with a suitable randomly generated $\hat{Q}_{pt}$. Then, we formulate the ISMOP for each time bucket $t = 1, \ldots, T$ and product cluster $g \in G$ as the following SLPP:

$$\min \sum_{p \in P} C_p f_{pt} \tag{7}$$

subject to

$$\frac{1}{|P_g|} \sum_{p \in P_g} \widehat{\mathrm{FFR}}_{pt}(f_{pt}, \hat{Q}_{pt}) \ge \mathrm{LoS}_g \tag{8}$$

$$f_{pt} = \lambda_p \mu_{pt} + k_p \sigma_{pt} \quad \forall p \in P_g \tag{9}$$

$$0 \le k_p \le k_p^{\max} \quad \forall p \in P_g \tag{10}$$