Energy Policy 36 (2008) 2911–2916

Contents lists available at ScienceDirect

Energy Policy

journal homepage: www.elsevier.com/locate/enpol

Linear programming models for measuring economy-wide energy efficiency performance

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ARTICLE INFO

Article history:
Received 5 December 2007
Accepted 25 March 2008
Available online 3 June 2008

Keywords:
Energy efficiency
Undesirable outputs
Data envelopment analysis

ABSTRACT

Data envelopment analysis (DEA) has recently gained popularity in energy efficiency analysis. A common feature of the previously proposed DEA models for measuring energy efficiency performance is that they treat energy consumption as an input within a production framework without considering undesirable outputs. However, energy use results in the generation of undesirable outputs as by-products of producing desirable outputs. Within a joint production framework of both desirable and undesirable outputs, this paper presents several DEA-type linear programming models for measuring economy-wide energy efficiency performance. In addition to considering undesirable outputs, our models treat different energy sources as different inputs so that changes in energy mix could be accounted for in evaluating energy efficiency. The proposed models are applied to measure the energy efficiency performances of 21 OECD countries and the results obtained are presented.

1. Introduction

With high energy prices and the concern about global warming and sustainable development, energy efficiency has become a vital part of the energy strategy in many countries (Ang, 2006). Researchers have developed appropriate indicators for monitoring economy-wide energy efficiency trends over time or comparing energy efficiency performances across countries/regions. A number of national energy agencies and international organizations have developed their energy efficiency measurement and monitoring systems. See, for example, IEA (2004a, 2007), EECA (2006), NRC (2006), OEERE (2007) and ODYSSEE (2007).

The foremost issue in the measurement of energy efficiency performance is to define the term "energy efficiency" (Patterson, 1996; Ang, 2006). There exist various definitions of energy efficiency, among which "the ratio of energy services to energy input" is a popular one. The definition given in the Directive 2006/32/EC of the European Council and the Parliament on energy end-use efficiency and energy services is a general one, namely energy efficiency is "a ratio between an output of performance, service, goods or energy, and an input of energy". Different definitions of energy efficiency would lead to different indicators being used to monitor changes in energy efficiency, which can yield very different results and policy implications (Berndt, 1978).[1] At the economy-wide level, since there is no single meaningful measure for energy services across all energy-consuming sectors and as such various approaches to measuring energy efficiency performance have been proposed in the literature.

A common practice to measure economy-wide energy efficiency performance is to first decompose the change in energy consumption or aggregate energy intensity into a number of contributing factors, and then aggregate the effects of energy intensity changes at energy end-use or sub-sector level to give a composite energy efficiency performance index (Ang, 2006). The decomposition of energy consumption or aggregate energy intensity can be implemented by the index decomposition analysis (IDA) technique (Ang and Zhang, 2000; Ang 2004; Liu and Ang, 2007). This IDA-based approach has been adopted by a number of countries including Canada, New Zealand and the United States to track economy-wide energy efficiency trends over time (EECA, 2006; NRC, 2006; OEERE, 2007).

It has been found that IDA-based energy efficiency studies mainly dealt with the measurement of energy efficiency changes over time in a specific entity, such as a country or a specific energy-consuming sector. Few of them dealt with the benchmarking of energy efficiency performance across different entities. In contrast, data envelopment analysis (DEA) has recently been

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[1] Patterson (1996) classified various energy efficiency indicators into thermodynamic, physical–thermodynamic, economic–thermodynamic and economic indicators. The DEA-related energy efficiency indexes proposed in this study and some earlier studies widen the scope further by modeling energy efficiency in a multiple inputs (inclusive of non-energy inputs) and multiple outputs production framework.

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doi:10.1016/j.enpol.2008.03.041

widely applied to evaluate the energy efficiency performances of different entities.

DEA, proposed by Charnes et al. (1978), is a well-established non-parametric frontier approach to evaluating the relative efficiency of a set of comparable entities featured with multiple inputs and outputs. The recent literature survey by Zhou et al. (2008a) found a rapid increase in the number of studies using DEA in the broad area of energy and environmental analysis. In energy efficiency studies, DEA has also gained in popularity. For instance, Boyd and Pang (2000) discussed the relationship between productivity and energy efficiency, and Ramanathan (2000) used DEA to compare the energy efficiencies of alternative transport modes. More recently, Onut and Soner (2006) applied DEA to assess the energy efficiencies of five-star hotels in Turkey. Hu and Wang (2006) and Hu and Kao (2007) developed a total-factor energy efficiency index by using DEA. Azadeh et al. (2007) proposed an integrated DEA approach to assessing the energy efficiency of energy-intensive manufacturing sectors. Wei et al. (2007) investigated the energy efficiency change of China's iron and steel sectors by using DEA-based Malmquist index approach. Mukherjee (2008) presented several DEA models for measuring the energy efficiency of manufacturing sectors. Lee (2008) combined regression analysis with DEA to study the energy efficiency of government buildings.

A common feature of the DEA models in the above-mentioned studies is that they model energy consumption as an input within a production framework where both energy and other non-energy inputs are used to produce good or desirable outputs. However, energy use also results in the generation of some undesirable outputs, e.g. CO_2 emissions, as by-products of producing desirable outputs. The measurement of energy efficiency without considering undesirable outputs does not seem to provide an equitable score for energy efficiency benchmarking and comparisons. It would therefore be appropriate to evaluate the economic-wide energy efficiencies within a joint production framework where both desirable and undesirable outputs are considered simultaneously.

This paper presents several DEA-type linear programming models within a joint production framework for measuring economy-wide energy efficiency performance. In addition to considering undesirable outputs, our models treat different energy sources as different inputs so that changes in energy mix could be accounted for in evaluating energy efficiency. The rest of this paper is organized as follows. Section 2 proposes the models for measuring energy efficiency performance. In Section 3, we present an empirical application study on measuring the energy efficiency performance of 21 OECD countries. Section 4 concludes this study.

2. Linear programming models for energy efficiency measurement

Consider a production process in which desirable and undesirable outputs are jointly produced by consuming both energy and non-energy inputs. Assume that **x**, **e**, **y** and **u** are, respectively, the vectors of non-energy inputs, energy inputs, desirable outputs and undesirable outputs, where energy inputs consist of L different energy sources.[2] Conceptually, the production technology for modeling the joint production process can be described as

$$T = \{(\mathbf{x}, \mathbf{e}, \mathbf{y}, \mathbf{u}) : (\mathbf{x}, \mathbf{e}) \text{ can produce } (\mathbf{y}, \mathbf{u})\} \tag{1}$$

In production theory, T is assumed to be a closed and bounded set, which guarantees the output closeness and implies that finite amounts of inputs can only produce finite amounts of outputs (Färe and Primont, 1995). In addition, inputs and desirable outputs in T are often assumed to be strongly disposable. Accordingly, if $(\mathbf{x},\mathbf{e},\mathbf{y},\mathbf{u}) \in T$ and $(\mathbf{x}',\mathbf{e}') \geqslant (\mathbf{x},\mathbf{e})$ (or $\mathbf{y}' \leqslant \mathbf{y}$) then $(\mathbf{x}',\mathbf{e}',\mathbf{y},\mathbf{u}) \in T$ (or $(\mathbf{x},\mathbf{e},\mathbf{y}',\mathbf{u}) \in T$).

In order to reasonably model the joint production of both desirable and undesirable outputs, following Färe et al. (1989), we impose the following two conditions on T:

(i) Outputs are weakly disposable, i.e., if $(\mathbf{x},\mathbf{e},\mathbf{y},\mathbf{u}) \in T$ and $0 \leqslant \theta \leqslant 1$, then $(\mathbf{x},\mathbf{e},\theta\mathbf{y},\theta\mathbf{u}) \in T$.
(ii) Desirable outputs and undesirable outputs are null-joint, i.e., if $(\mathbf{x},\mathbf{e},\mathbf{y},\mathbf{u}) \in T$ and $\mathbf{u} = 0$, then $\mathbf{y} = 0$.

The first condition implies that the reduction of undesirable outputs is not free but the proportional reduction in both desirable and undesirable outputs is feasible. The second condition implies that the only way to eliminate all the undesirable outputs is to cease the production process.

Although the production technology T has been well defined for modeling the joint production of desirable and undesirable outputs, it cannot be directly used in empirical studies. In application, a popular practice is to formulate it within a non-parametric DEA framework. The resulting technology could therefore be termed as an environmental DEA technology (Färe and Grosskopf, 2004; Zhou et al., 2008b). In the case where there are K entities whose energy efficiency performances are to be measured, and for the kth entity the observed data on non-energy inputs, energy inputs, desirable and undesirable outputs are $\mathbf{x}_k = (x_{1k},\ldots,x_{Nk})$, $\mathbf{e}_k = (e_{1k},\ldots,e_{Lk})$, $\mathbf{x}_k = (x_{1k},\ldots,x_{Nk})$, $\mathbf{y}_k = (y_{1k},\ldots,y_{Mk})$ and $\mathbf{x}_k = (x_{1k},\ldots,x_{Nk})$, $\mathbf{u}_k = (u_{1k},\ldots,u_{Jk})$, the environmental DEA technology exhibiting constant returns to scale (CRS) can be expressed as

$$\begin{aligned} T = \{(\mathbf{x}, \mathbf{e}, \mathbf{y}, \mathbf{u}) : & \sum_{k=1}^{K} z_k x_{nk} \leqslant x_n, \quad n = 1, \ldots, N \\ & \sum_{k=1}^{K} z_k e_{lk} \leqslant e_l, \quad l = 1, \ldots, L \\ & \sum_{k=1}^{K} z_k y_{mk} \geqslant y_m, \quad m = 1, \ldots, M \\ & \sum_{k=1}^{K} z_k u_{jk} = u_j, \quad j = 1, \ldots, J \\ & z_k \geqslant 0, \quad k = 1,\, 2, \ldots, K\} \end{aligned} \tag{2}$$

It can be easily verified that the CRS environmental DEA technology, i.e. (2), satisfies all the conditions mentioned above. In empirical studies, the CRS environmental DEA technology integrated with various efficiency measures has been widely used in diverse areas, such as productivity estimation with pollutants considered and environmental performance measurement. Examples of such studies include Tyteca (1996), Boyd and

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[2] Most traditional energy efficiency indicators consider only energy inputs in energy efficiency assessment (Patterson, 1996). However, as discussed by Hu and Wang (2006), energy inputs alone cannot produce any outputs without incorporating other non-energy inputs. It would therefore be appropriate to consider both inputs together in assessing energy efficiency within a production framework. In the literature, this line of reasoning has been adopted by most DEA-

(*footnote continued*)
related energy efficiency studies, e.g. Hu and Wang (2006), Onut and Soner (2006), Hu and Kao (2007), and Mukherjee (2008). Compared with the traditional energy efficiency indicators such as the energy intensity where only energy inputs are considered, DEA-based energy efficiency indexes could be treated as total-factor energy efficiency or productivity indexes.