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A linear programming model for determining ordered weighted averaging operator weights with maximal Yager's entropy

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ABSTRACT

It has a wide attention about the methods for determining OWA operator weights. At the beginning of this dissertation, we provide a briefly overview of the main approaches for obtaining the OWA weights with a predefined degree of orness. Along this line, we next make an important generalization of these approaches as a special case of the well-known and more general problem of calculation of the probability distribution in the presence of uncertainty. All these existed methods for dealing these kinds of problems are quite complex. In order to simplify the process of computation, we introduce Yager's entropy based on Minkowski metric. By analyzing its desirable properties and utilizing this measure of entropy, a linear programming (LP) model for the problem of OWA weight calculation with a predefined degree of orness has been built and can be calculated much easier. Then, this result is further extended to the more realistic case of only having partial information on the range of OWA weights except a predefined degree of orness. In the end, two numerical examples are provided to illustrate the application of the proposed approach.

1. Introduction

The process of information aggregation has a great affect on the development of intelligent systems. Yager (1988) introduced a new information aggregation technique based on the ordered weighted averaging (OWA) operator. One key issue in the theory of the OWA operator is to determine its associated weights (Amin, 2006, 2007; Ahn, 2006; Filev & Yager, 1998; Nettleton & Torra, 2001; Wang Y, Luo, & Liu, 2007). O'Hagan (1988) proposed a maximum entropy approach, which involved a constrained nonlinear optimization problem with a predefined degree of orness as its constraint and the entropy as the objective function. Then, Fullér and Majlender (2001) transformed the maximum entropy model into a polynomial equation which can be solved analytically. Fullér and Majlender (2003) suggested a minimum variance approach to obtain the minimal variability OWA operator weights. Majlender (2005) proposed an approach for obtaining OWA operator weights based on maximal Rényi entropy for a given level of orness and pointed out that the maximum entropy approach and the minimum variance approach are its special cases, respectively. Liu and Chen (2004) proposed the PMEOWA operator and Liu (2006) proposed the MSEOWA operator. Few scholars study the nature of these models and the relationship between them. Moreover, there are two main shortcomings with in all the above approaches: (1) Generally, it is quite complex to acquire the solution of a constrained nonlinear optimization problem or a high-order nonlinear algebraic equation (Wang, 2005). (2) All of them are completely based on the assumption of given orness level. As a fact, it may be difficult for the decision maker (DM) to determine his/her orness in some circumstances. For instance, Xu and Da (2003) established an approach by considering the situation where partial weight information is available partially. The DM may also have other type of weight information except a predefined degree of orness (Kim & Ahn, 1999; Park & Kim, 1997).

In order to simplify the complicated computation, we introduce Yager's entropy based on Minkowski metric (Yager, 1995). By analyzing the desirable properties with this measure of entropy, we propose a novel approach to determine the weights of the OWA operator. It is a wide general method which can be specialized into many famous cases, such as the minimax model (Yager, 1993), the MSEOWA operator (Liu, 2006) and the minimum variance approach (Fullér & Majlender, 2003). We then extend it to a linear objective programming (LP) model with a predefined degree of orness. Further, we consider the (LP) model for the more realistic case, which miss a priori information for the desired orness and only has partial weight information as constraint.

The dissertation is organized as follows. Section 2 gives a brief overview and makes an important generalization of the main approaches for OWA weights with a priori information for the desired orness. Section 3 proposes a LP model for OWA weights based on Yager's entropy. In Section 4, two numerical examples are provided. Section 5 concludes the paper.

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2. The overview and generalization of the methods for OWA weights

An OWA operator of dimension n is a mapping, OWA : $R^n \to R$, that has an associated n vector $w = (w_1, w_2, \ldots, w_n)^T$ such that $w_j \in [0,1]$ and $\sum_{j=1}^n w_j = 1$. Furthermore,

$$\mathrm{OWA}(a_1, a_2, \ldots, a_n) = \sum_{j=1}^n w_j b_j \quad (1)$$

where b_j is the jth largest element of the collection of the aggregated objects $a_1, a_2, \ldots, a_n$.

Clearly, one key point of the OWA operator theory is to determine its associated weights. O'Hagan (1988) suggested a maximum entropy approach, which requires the solution of the following constrained nonlinear optimization problem:

$$\begin{aligned} &\text{Max disp}(W) = -\sum_{i=1}^n w_i \ln w_i \\ &\text{s.t. orness}(W) = \sum_{i=1}^n \frac{n-i}{n-1} w_i = \alpha, \quad 0 \leqslant \alpha \leqslant 1 \\ &\sum_{i=1}^n w_i = 1, \quad w_i \in [0,1], \; i = (1, \ldots, n) \end{aligned} \quad (2)$$

The weight resolved from model (2) is called that maximum entropy OWA (MEOWA) weights.

Liu and Chen (2004) proposed the PMEOWA operator. The problem can be formulated as:

$$\begin{aligned} &\text{Max disp}(W) = -\sum_{i=1}^n w_i \ln w_i \\ &\text{s.t.} \; \sum_{i=1}^n x_i w_i = c, \\ &\sum_{i=1}^n w_i = 1, \quad w_i \in [0,1], \; i = (1, \ldots, n) \end{aligned} \quad (3)$$

where $x_i > x_j$ for $i < j (i, j = 1, 2, \ldots, n)$ and $x_i > c > x_j$.

Fullér and Majlender (2003) suggested a minimum variance approach, which minimizes the variance of OWA operator weights under a given level of orness. A set of OWA operator weights with minimal variability could then be generated. Their approach requires the solution of the following mathematical programming problem:

$$\begin{aligned} &\text{Min } D^2(W) = \frac{1}{n} \sum_{i=1}^n \left(w_i - \frac{1}{n} \right)^2 = \frac{1}{n} \left(\sum_{i=1}^n w_i^2 - \frac{1}{n} \right) \\ &\text{s.t. orness}(W) = \sum_{i=1}^n \frac{n-i}{n-1} w_i = \alpha, \quad 0 \leqslant \alpha \leqslant 1 \\ &\sum_{i=1}^n w_i = 1, \quad w_i \in [0,1], \; i = (1, \ldots, n) \end{aligned} \quad (4)$$

Recently, Majlender (2005) proposed an approach for obtaining OWA operator weights based on maximal Rényi entropy for a given level of orness. This approach is based on the solution of the following parametric mathematical programming problem:

$$\begin{aligned} &\text{Max } H_c(W) = \frac{1}{1-c} \log_2 \sum_{i=1}^n w_i^c = \log_2 \left(\sum_{i=1}^n w_i^c \right)^{1/(1-c)} \\ &\text{s.t. orness}(W) = \sum_{i=1}^n \frac{n-i}{n-1} w_i = \alpha, \quad 0 \leqslant \alpha \leqslant 1 \\ &\sum_{i=1}^n w_i = 1, \quad w_i \in [0,1], \; i = (1, \ldots, n) \end{aligned} \quad (5)$$

From strict monotonicity of the logarithm function, Majlender proved that model (5) includes models (2) and (4) as its special cases for $c = 1$ and $c = 2$, respectively.

To discuss the relationships in the above models, Wu, Liang, and Huang, 2007 considered the models (2)–(5) as a method for setting up faire probability distributions on the basis of partial information. For analyzing the probability distributions, we shall introduce the principle of maximum entropy proposed by Jaynes (1957), which has the important property that no possibility is ignored and assigns positive weight to every situation that is not absolutely excluded by the given information.

The quantity$x$ is capable of assuming the discrete values $x_i (i = 1, 2, \ldots, n)$. We are not given the corresponding probabilities p_i; all we know is $\langle f_r(x) \rangle$, which is the expectation value of the function $f_r(x)$. To determine the probabilities p_i fairly, Jaynes proposed the optimization problem:

$$\begin{aligned} &\text{Max } H(p_1, \ldots, p_n) = -\sum_{i=1}^n p_i \ln p_i \\ &\text{s.t.} \; \langle f_r(x) \rangle = \sum_{i=1}^n p_i f_r(x_i) \\ &\sum_{i=1}^n p_i = 1, \quad p_i \geqslant 0; \; r = 1, \ldots, m; \; i = 1, \ldots, n \end{aligned} \quad (6)$$

To resolve model (6), we introduce Lagrangian Multipliers $\lambda_0, \lambda_1, \ldots, \lambda_r, \ldots, \lambda_m$, in the usual way, and obtain the result

$$p_i = \exp\{-[\lambda_0 + \lambda_1 f_1(x_i) + \cdots + \lambda_m f_m(x_i)]\} \quad (7)$$

in which the constants $\lambda_0, \lambda_1, \ldots, \lambda_r, \ldots, \lambda_m$ are determined from

$$Z(\lambda_1, \ldots, \lambda_m) = \sum_{i=1}^n \exp\{-[\lambda_1 f_1(x_i) + \cdots + \lambda_m f_m(x_i)]\} \quad (8)$$

$$\langle f_r(x) \rangle = -\frac{\partial}{\partial \lambda_r} \ln Z(\lambda_1, \ldots \lambda_r \ldots, \lambda_m) \quad (9)$$

$$\lambda_0 = \ln Z(\lambda_1, \ldots \lambda_r \ldots, \lambda_m) \quad (10)$$

Especially when $f_r(x_i) = 0, r = 2, \ldots, m; i = 1, \ldots, n$, we can get

$$\begin{aligned} &\text{Max } H(p_1, \ldots, p_n) = -\sum_{i=1}^n p_i \ln p_i \\ &\text{s.t.} \; \langle f_1(x) \rangle = \sum_{i=1}^n p_i f_1(x_i) \\ &\sum_{i=1}^n p_i = 1, \quad p_i \geqslant 0; \; i = 1, \ldots, n \end{aligned} \quad (11)$$

Obviously, model (11) includes models (2) and (4) as its special cases for $f_1(x_i) = \frac{n-i}{n-1}$ and $f_1(x_i) = x_i$, respectively. So we can find that model (2) and model (4) satisfy the principle of maximum entropy. But the above approaches are quite complex for requiring the solution of a constrained nonlinear optimization problem. Furthermore, the computational burden of models (2)–(5) will increase when the DM put more information into these models. To resolve this problem, this paper will propose a linear objective programming (LP) model for determining OWA operator weights by utilizing the Minkowski metric-based measures of entropy introduced by Yager.

3. LP model for OWA weights based on Yager's entropy

3.1. Yager's entropy based on minkowski metric and its properties

Yager (1995) developed a measure of neg-entropy as:

$$Q_Z(P) = D\left(P, \left[\frac{1}{n}\right]\right) = \left(\sum_{i=1}^n \left| p_i - \frac{1}{n} \right|^Z \right)^{1/Z}, \quad Z \geqslant 1, \; i = 1, 2, \ldots, n \quad (12)$$

where the vector $[1/n]$ is the probability distribution $p_i = 1/n$ and $[I_j]$ denotes the probability distribution with $p_j = 1$. It is easy to see that $Q_Z(I_i) = Q_Z(I_j) = Q_Z(I)$, the distance to any singleton is the