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Comparative calculation of irrigation networks using Labye's method, the linear programming method and a simplified nonlinear method

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ABSTRACT

The designating factors in the design of branched irrigation networks are the cost of pipes and the cost of pumping. They both depend directly on the hydraulic pump head. It is mandatory for this reason to calculate the optimal pump head as well as the corresponding economic pipe diameters, in order the minimal total cost of the irrigation network to be produced. The classical optimization techniques, which have been proposed so long, are the following: the linear programming optimization method, the nonlinear programming optimization method, the dynamic programming optimization method and Labye's method. The mathematical research of the problem using the above classical optimization techniques is very complex and the numerical solution calls for a lot of calculations, especially in the case of a network with many branches. For this reason, many researchers have developed simplified calculation methods with satisfactory results and with less calculation time needed. A simplified nonlinear optimization method has been developed at the Aristotle University of Thessaloniki — Greece by M. Theocharis. The required calculating procedure is much shorter when using Theocharis' simplified method than when using the classic optimization methods, because Theocharis' method requires only a handheld calculator and just a few numerical calculations. In this paper a comparative calculation of the pump optimal head as well as the corresponded economic pipe diameters, using: (a) Labye's optimization method, (b) the linear programming optimization method and (c) Theocharis' simplified nonlinear programming method is presented. Application and comparative evaluation in a particular irrigation network is also developed. From the study it is concluded that Theocharis' simplified method can be equally used with the classical methods.

1. Introduction

The problem of selecting both, the best arrangement for the pipe diameters and the optimal pump head, so that the minimal total cost of a project to be produced, received considerable attention many years ago by the engineers who study hydraulic works. The classical optimization techniques, which have been proposed so far, are the following: (a) The linear programming optimization method [1–8], (b) the nonlinear programming optimization method [4,5,8,9], (c) the dynamic programming method [4,8], and (d) Labye's optimization method [4,7,8,10–14]. The common characteristic of all the above

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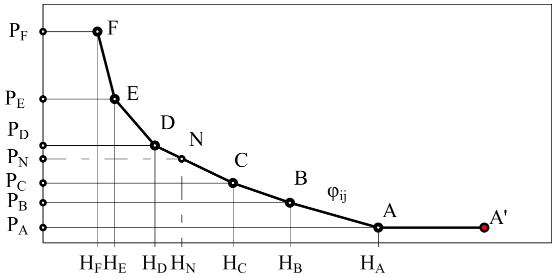

Fig. 1. A network with pipes in sequence.

techniques is an objective function, which includes the total cost of the network pipes, and which is optimized according to specific constraints. The decision variables that are generally used are: the pipe diameters, the head losses, and the pipe lengths. As constraints are used: the pipe lengths, and the available piezometric heads in order to cover the friction losses. The mathematical research of the problem using the above classical optimization techniques is very complex and the numerical solution calls for a lot of calculations, especially in the case of a network with many branches. A simplified nonlinear programming optimization method which does not require the use of computers is based on the observation [4,6,15,16] that each supplied branch of a branched network tends to raise the economic hydraulic gradient at the junction points, above the economic hydraulic gradient of the network, which does not include this branch. Thus, if the complete route of the network presenting the minimum average hydraulic gradient is selected, the economic piezometric line corresponding to this (assuming that all the other supplied branches are neglected) tends to be raised under the influence of the supplied branches, which at first had been neglected, so that the final hydraulic gradient does not differ remarkably from this economic hydraulic gradient. But only one equation is needed to calculate the economic hydraulic gradient corresponding to the complete route of the network presenting the minimum average gradient. Using the piezometric heads at the junction points, which have been calculated by the above-mentioned procedure from the complete route presenting the minimum average gradient, as heads of the supplied branches, the frictional head losses and the diameters of the pipes within the supplied branches can be easily calculated.

In the present work a brief presentation of: (a) Labye's optimization method, (b) the linear programming optimization method and (c) Theocharis' simplified nonlinear programming optimization method, is presented. Application and comparative evaluation in a particular irrigation network are also developed. The results of this comparison show that the total annual cost of the project is in fact the same in any case. So Theocharis' simplified method can be applied with no distinction in the study of the branched hydraulic networks.

2. Methods

2.1. Labye's optimization method

According to this method [4,7,8,10–14], the optimal solution of hydraulic networks is obtained considering that the pipe diameters can only be chosen in a discrete set of values corresponding to the standard ones considered. It consists of the tracing of a zigzag line in a coordinates diagram, from which the minimal cost of the network can be obtained as a function of the total piezometric losses of the network.

2.1.1. A network with pipes in sequence

For every pipe of the network [4,7,10,14] the available commercial sizes of diameters are selected and then calculated: i. the frictional head losses per meter, J_{ij}; ii. the pipe cost per meter, c_{ij}; and iii. the various gradients $\varphi_{ij} = |\frac{\Delta c_{ij}}{\Delta J_{ij}}|$ which are classified in decreased order. After that, the graph P–H, (Fig. 1) is constructed which is a convex zigzag line called "*the characteristic*" of the network. The gradients of the zigzag line various parts, φ_{ij}, are progressively decreased from left to right. The terminal right point of the characteristic, A, corresponds to the minimal diameters for all the pipes of the network with the maximal total frictional losses, H_A, and the minimal total cost of the network P_A. Similarly the terminal left point of the characteristic, F, corresponds to the maximal diameters for all the pipes of the network with the minimal total frictional losses, H_F, and the maximal total cost of the network P_F.

After that, the total cost of the network, P_N, corresponding to the available total frictional head loss, H_N, is calculated. If the point N lies in the part of the characteristic with gradient φ_{ij}, it means that only the ith pipe must be constructed with two different diameters. On the passing from the point N to the point F, linear parts with progressively increasing gradients corresponding to the various pipes of the network are detected. Each pipe is constructed with the lower diameter corresponding to the gradient φ_{ij}. Similarly any pipe, the gradients of which are detected on the right of the point N, is constructed with the higher diameter corresponding to the gradient φ_{ij}. It is concluded that only one pipe of the network is possible to be constructed with two different diameters.