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Solving multi-choice linear programming problems by interpolating polynomials

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ABSTRACT

Multi-choice programming solves some optimization problems where multiple information exists for a parameter. The aim of this paper is to select an appropriate parameter from a set of multiple choices, which optimizes the objective function. We consider a linear programming problem where the right hand side parameters are multi-choice in nature. In this paper, the multiple choices of a parameter are considered as functional values of an affine function at some non-negative integer nodes. An interpolating polynomial is formulated using functional values at non-negative integer nodes to take care of any multi-choice parameter. After establishing interpolating polynomials of all multi-choice parameters, a mathematical programming problem is formulated. The formulated problem is treated as a nonlinear programming problem involving mixed integer type variables. It can be solved by using standard nonlinear programming software. Finally, a numerical example is presented to illustrate the solution procedure.

1. Introduction

In Operations Research (OR), a real life decision making problem is transformed into a mathematical model that attempts not only to explain the behavior of the system but also to find an optimal solution of that system. A Mathematical Programming (MP) problem is an optimization problem where we maximize/minimize one or more mathematical functions, known as objective functions subject to certain constraints imposed on the problem, which are represented by mathematical equations or inequalities with some restrictions on the decision variables. An MP problem is often extended over a parameter space p.

The mathematical model of such an optimization problem can be represented as

$$\max / \min : f(X; p) \tag{1.1}$$

subject to

$$g(X; p) \leq 0 \tag{1.2}$$
$$X \geq 0 \tag{1.3}$$

where $X \subset R^n$ and the functions f and g are defined as $f, g : R^n \to R$.

If we consider the linearity of the functions f and g, then the MP problem can be divided into two different problems, namely, linear and nonlinear optimization problems. Similarly, some known subsets of the MP problem are called constrained and unconstrained optimization (depending on the presence or absence of constraints), single objective and

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multi-objective optimization (depending on one or more objective functions), continuous, discrete, and mixed integer optimization (depending on the nature of decision variables), and deterministic, fuzzy, and stochastic optimization (depending on the parameter space). It has been observed that, the parameters that form the parameter space may be of multi-choice type, i.e., there may exist a set of choices for a parameter, out of which only one is to be selected to optimize the objective function(s). Such a type of MP problem is known as multi-choice optimization.

Multi-choice optimization problems occur in real life in different situations, namely, while selecting a new car, selecting new security personnel, employing a number of sales persons in a given period for a super market, implementing a new policy for a community etc.

2. Literature review

The Multiple Choice Programming (MCP) problem, which is originated by Healey [1], belongs to a class of combinatorial optimization problems with a requirement to choose, among several possible combinations as an alternative to optimize an objective function subject to a set of constraints. In practice, MCP can be extended as an application of generalized assignment problems, multiple choice knapsack problems, sales resource allocation, multi-item scheduling, timetabling, etc. MCP is a mixed binary programming where all binary variables constitute a number of mutually exclusive choices where only one variable is to be selected.

The state-of-the-art survey pertaining to MCP has been reported by Snyder [2] and Lin [3]. The situation of multiple choices for a parameter exists in many managerial decision making problems. In this paper, some multi-choice parameters are considered in a linear programming problem. A decision maker (DM) is allowed to set multiple number of choices for a given parameter. Ravindran et al. [4] and Hiller and Lieberman [5] have considered a mathematical model in which an appropriate constraint is to be chosen using binary variables. The number of binary variables required for a constraint is same as the total number of choices of that constraint. Chang [6] has proposed the formulation of multi-choice goal programming (MCGP), which allows the DMs to set multi-choice aspiration levels (MCAL) for each goal to avoid the underestimation of decision making. He used multiplicative terms of binary variables to handle the multiple aspiration levels. The transformed mathematical programming model proposed by Chang [6] is easily implementable by a DM when the number of aspiration levels assigned to a goal is an integral power of 2. When the number of aspiration levels is not an integral power of 2, the model formulation is a difficult task. In his other paper [7], multiplicative terms of the binary variables are replaced by continuous variables. Liao [8] proposed a method to solve the multi-segment goal programming problem, which finds a solution close to the DM's multi-segment aspiration levels. He has tackled the multi-choice parameters the way which was proposed by Chang [6]. He suggested a method where a multi-choice parameter accommodates at best three choices. Biswal and Acharya [9] proposed a transformation technique for a multi-choice linear programming (MCLP) problem which accommodates at best eight choices for a goal. Some auxiliary constraints have been used to avoid repetition. Some authors, namely [10,11] have applied the multi-choice goal programming approach of Chang [6] to solve some real life decision making problems. Further, Paksoy and Chang [12] have applied the revised multi-choice goal programming approach of Chang [7] in order to deal with multi-choice parameters to solve a supply chain network design problem.

We face the following major difficulties when we transform an MCLP problem into a standard mathematical programming problem involving more number of choices for a given parameter:

(i) selecting binary variables,
(ii) selecting bounds for binary codes,
(iii) restricting binary codes using auxiliary constraints.

In order to avoid such difficulties, we apply some theory of numerical methods, namely, the interpolating polynomial method for a multiple choice parameter. Interpolating polynomials are formulated for all the multi-choice parameters of the problem. The multi-choice parameters are replaced by corresponding interpolating polynomials to formulate a mixed integer nonlinear programming problem, which can be solved by a nonlinear programming technique.

The organization of the paper is as follows: after the introduction and literature review a mathematical model of MCLP problem is presented in Section 3. Formulations of equivalent mathematical programming models under different types of interpolating polynomials are presented in Section 4. In order to demonstrate the proposed method a case study is included in Section 5. Finally, discussion of results and conclusions are presented in Section 6.

3. Mathematical model of an MCLP problem

The mathematical model of an MCLP problem is presented as follows.

Find $X = (x_1, x_2, x_3, \ldots, x_n)$ so as to

$$\max : Z = \sum_{j=1}^{n} c_j x_j \tag{3.4}$$