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Solving optimal growth models with vintage capital: The dynamic programming approach ☆

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Abstract

This paper deals with an endogenous growth model with vintage capital and, more precisely, with the AK model proposed in [R. Boucekkine, O. Licandro, L.A. Puch, F. del Rio, Vintage capital and the dynamics of the AK model, J. Econ. Theory 120 (1) (2005) 39–72]. In endogenous growth models the introduction of vintage capital allows to explain some growth facts but strongly increases the mathematical difficulties. So far, in this approach, the model is studied by the Maximum Principle; here we develop the Dynamic Programming approach to the same problem by obtaining sharper results and we provide more insight about the economic implications of the model. We explicitly find the value function, the closed loop formula that relates capital and investment, the optimal consumption paths and the long run equilibrium. The short run fluctuations of capital and investment and the relations with the standard AK model are analyzed. Finally the applicability to other models is also discussed.

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MSC: 49L20; 49K25; 34K35

Keywords: Endogenous growth; Vintage capital; AK model; Dynamic programming

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0. Introduction

In this work we develop the Dynamic Programming (DP in the following) approach to study a continuous time endogenous growth model with vintage capital. We focus on the AK model proposed by Boucekkine, Puch, Licandro and Del Rio in [14] (see e.g. [11], [8] for related models) which is summarized in Section 1.1.

In the literature continuous time endogenous growth models with vintage capital are treated by using the Maximum Principle (MP in the following). Here we develop the DP approach to the representative model of [14] getting sharper results. The improvements we obtain mainly come from the fact that we are able to find the value function and solve the optimal control problem in closed loop form, a key feature of the DP approach.

We stress the fact that the novelty of this paper is mainly on the methodological side.

In our opinion the DP approach to continuous time optimal control problems arising in economic theory has not been exploited in its whole power. This is especially true when the model presents some features (like the presence of Delay Differential Equations and/or Partial Differential Equations and/or state–control constraints) that call for the use of infinite dimensional analysis making it harder to treat with the standard theory. However the presence of such features is needed when we want to look at problems with vintage capital, see for instance the quoted papers [14,11,8], and also [7], [32], [30,31] on optimal technology adoption and capital accumulation.

To be clear and honest we must say that in this paper the DP approach works very well thanks to the availability of explicit solutions which happens also in other models (see Section 4.1) but when explicit solutions are not available still something interesting (like the points (II) and (III) below or the qualitative behavior of optimal path) can be said, usually with more technical difficulties. What can be said and the amount of difficulties strongly depend on the structure of the problem under study: in some cases almost everything can be repeated, in some other ones almost nothing, at the present stage (see Section 4.2). We also clarify that we are not saying that the DP is generally better than the MP approach: when the difficulties are hard it is often useful to use an integrated approach developing both the MP and the DP.[1] In Section 4 we present a detailed discussion on these points.

The main methodological issues treated in this paper are the following.

(I) (*Explicit form of solutions*).
Providing solutions in explicit form, when possible, helps the analysis of the model. In [14] it is shown that the optimal consumption path has a specific form (i.e. it is an exponential multiplied by a constant Λ) but none is said about the form of Λ, the explicit expression of the capital stock and investment trajectories. Moreover existence of a long run equilibrium for the discounted paths is established but none is said about its form.
Here, using the fact that we can calculate explicitly the value function, we show a more precise result on the optimal consumption path determining the constant Λ and an equation for the optimal trajectories of the capital stock and of the investment. This allows to find explicitly the long run equilibrium of the discounted paths; in particular we can give more precise analysis of the presence of oscillations in the capital and investment stock and in the

[1] For example such an integrated approach is used successfully in [34].