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Complexity-performance trade-off of algorithms for combined lattice reduction and QR decomposition☆

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ABSTRACT

Over the last years, numerous equalization schemes for multiple-input/multiple-output channels have been studied in the literature. New low-complexity approaches based on lattice basis reduction are of special interest, since they achieve the optimum diversity behavior. Although the per-symbol equalization complexity of these schemes is very low, the initial calculation of the required matrices may impose an enormous burden in arithmetic complexity. In this paper, we give a tutorial overview and assess algorithms, which, given the channel matrix, result in the feedforward, feedback, and unimodular matrix required in lattice-reduction-aided decision-feedback equalization or precoding. To this end, via a unified exposition of the Lenstra–Lenstra–Lovász (LLL) algorithm, the LLL with deep insertions, and the reversed Siegel approach similarities and differences of these approaches are enlightened. A modification of the LLL swapping criterion, better matched to the equalization setting, is discussed. It is shown that using lattice-reduction-aided equalization/precoding better performance can be achieved at lower complexity compared to classical equalization or precoding approaches.

1. Introduction

Communication using antenna arrays at transmitter and receiver, hence creating a multiple-input/multiple-output (MIMO) channel, is very interesting because of the very high spectral efficiencies and hence data rates which can be achieved, cf. [33,13,27]. The multi-antenna interference present in such a scenario has to be combatted with some means of equalization.

The same situation occurs when considering multi-user transmission schemes, where multiple, usually non-cooperating users send their data streams over a common channel hence creating multi-user interference. Here, the task of a joint receiver is to separate the users, i.e., to equalize the channel, which again can be described by a MIMO channel matrix.

Taking the uplink/downlink duality [22,28] into account, instead of employing receiver-side equalization for the MIMO channel pre-equalization or precoding can be performed. In point-to-point transmission schemes transmitter- or receiver-side techniques can be used alternatively, whereas in a multi-user uplink (downlink) only receiver-side (transmitter-side) equalization is possible.

Over the last years, a number of equalization schemes for MIMO channels has been studied in the literature. Numerous techniques known from intersymbol-interference channels (e.g., linear (pre-)equalization, decision-feedback equalization (DFE),[1] Tomlinson–Harashima precoding (THP), maximum-likelihood detection (MLD), vector precoding, cf. [7, Table E.1], see also [38]) have been transferred to the MIMO setting. However, new approaches based on lattice basis reduction, e.g., [37,30,32,35], are of special interest. For an overview on lattice reduction see [36]. Using these lattice-reduction-aided (LRA) techniques, low-complexity equalization achieving the optimum diversity behavior is enabled [26]. Although the per-symbol equalization complexity is very low when applying LRA techniques the initial calculation of the required matrices still imposes an enormous burden in arithmetic complexity.

In this paper, we study algorithms for calculating the matrices to be used in LRA DFE or LRA precoding. As the structure and the per-symbol processing of the equalization scheme itself is always the same, we restrict ourselves to the complexity required for performing the initial factorization task: given the channel matrix, what complexity is required to calculate the feedforward, feedback, and unimodular matrix. Conventional approaches perform the calculation of these matrices in two separated steps: first, a lattice basis reduction algorithm is applied to the channel matrix; second,

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[1] DFE is also known as successive cancellation and employed in the VBLAST (vertical bell laboratories layered space-time) scheme [13,14].

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a sorted QR decomposition of the reduced channel matrix is carried out.

We compare algorithms which performs these two steps in a joint fashion. To this end, complex-valued variants of the LLL algorithm—the original approach [16], the deep LLL [21], and the reversed Siegel algorithm [4]—whose internal results can be used readily (cf. [35]) are presented in a unified way. We particularly assess the tradeoff between computational complexity of these algorithms and the signal-to-noise ratio required to guarantee a desired error rate. It turns out that by appropriately adjusting the free parameter of the algorithms, an excellent tradeoff can be achieved—almost the performance of the optimum LRA schemes can be obtained with only marginally larger complexity than sorted QR decomposition.

The paper is organized as follows: in Section 2 the channel model and a brief review on lattice-reduction-aided decision-feedback equalization and precoding are given. Section 3 discusses algorithms from the literature in a unified way and presents some modifications. Numerical results are given and discussed in Section 4; Section 5 draws some conclusions.

2. Channel model and lattice-reduction-aided equalization

2.1. MIMO channel model

We consider multiple-antenna transmission over flat-fading channels where joint equalization is either possible at the receiver (uplink situation) or the transmitter side (downlink). The input/output relation in complex base-band notation is given by the usual equation

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}, \tag{1}$$

where[2] $\boldsymbol{x} = [x_1, \ldots, x_{N_\mathrm{T}}]^\mathrm{T}$ is the transmit vector of dimension N_T and $\boldsymbol{H} = [h_{l,k}]$ is the $N_\mathrm{R} \times N_\mathrm{T}$ MIMO channel matrix composed of the fading coefficients $h_{l,k}$. The N_R receive symbols are collected in the receive vector $\boldsymbol{y} = [y_1, \ldots, y_{N_\mathrm{R}}]^\mathrm{T}$, and $\boldsymbol{n} = [n_1, \ldots, n_{N_\mathrm{R}}]^\mathrm{T}$ is the noise vector; spatially white zero-mean noise with variance σ_n^2 per component, i.e., $\mathrm{E}\{\boldsymbol{n}\boldsymbol{n}^\mathrm{H}\} = \sigma_n^2 \boldsymbol{I}$, is assumed.

In case of receiver-side equalization, the transmit symbols are immediately given by the data symbols, i.e., $x_k = a_k$, $k = 1, 2, \ldots, N_\mathrm{T}$, taken from an M-ary square QAM signal constellation $\mathcal{A}$ (variance $\sigma_a^2 = \mathrm{E}\{|a_k|^2\}$, $\forall k$). If precoding is used, the transmit symbols are jointly calculated from the N_R data symbols dedicated to the individual users.

Even though all subsequent derivations are done based on the complex-valued channel model, they are equally valid for the equivalent real-valued model of doubled dimensionality [27]. The differences between both views and their advantages will be discussed later on.

2.2. Lattice-reduction-aided equalization

Lattice-reduction-aided (LRA) equalization and precoding schemes, e.g., [37,30,32,35,25,36], have proven to require only low complexity, nevertheless being able to achieve the full diversity order of the MIMO channel [26]. The idea is to choose a "more suited" representation of the lattice spanned by the columns/rows of the channel matrix $\boldsymbol{H}$; equalization is done with respect to the new, so-called reduced, basis, which is desired to be close to orthogonal. At the very end/very beginning, the change of basis is reversed.

For performing LRA equalization directly based on the complex-valued channel matrix $\boldsymbol{H}$, in the first step a complex-valued version (e.g., [2,12]) of the LLL algorithm [16] is used to perform lattice basis reduction. When desiring equalization optimized according to the minimum mean-squared error (MMSE) criterion, the augmented channel matrix $\boldsymbol{H}_{:}$ of dimension $(N_\mathrm{R} + N_\mathrm{T}) \times N_\mathrm{T}$ ($\zeta = \sigma_n^2/\sigma_a^2$ is the inverse signal-to-noise ratio (SNR)) has to be factorized according to

$$\boldsymbol{H}_{:} = \begin{bmatrix} \boldsymbol{H} \\ \sqrt{\zeta}\boldsymbol{I} \end{bmatrix} = \boldsymbol{C}\,\boldsymbol{T} = \begin{bmatrix} \boldsymbol{C}_\mathrm{u} \\ \boldsymbol{C}_\mathrm{l} \end{bmatrix} \boldsymbol{T} \ . \tag{2}$$

Thereby, $\boldsymbol{T}$ is a matrix which elements are Gaussian integers[3] and $|\det(\boldsymbol{T})| = 1$; in turn $\boldsymbol{T}^{-1}$ also contains only Gaussian integers.

Now, instead of performing equalization of the entire channel $\boldsymbol{H}$, only $\boldsymbol{C}$ is equalized which causes less noise enhancement. Thereby $\boldsymbol{T}\boldsymbol{a}$ is recovered. Since $\boldsymbol{T}\mathbb{G}^{N_\mathrm{T}} = \mathbb{G}^{N_\mathrm{T}}$, the elements of this vector are also drawn from the Gaussian integer lattice.[4] Via $\boldsymbol{T}^{-1}$ estimates $\hat{a}_k$ of the initial data symbols are generated.

In the second step, for performing LRA DFE the reduced channel matrix $\boldsymbol{C}$ is further decomposed according to [31]

$$\begin{bmatrix} \boldsymbol{C}_\mathrm{u} \\ \boldsymbol{C}_\mathrm{l} \end{bmatrix} \boldsymbol{P} = \boldsymbol{C}\boldsymbol{P} = \boldsymbol{Q}\boldsymbol{R} = \begin{bmatrix} \boldsymbol{Q}_\mathrm{u} \\ \boldsymbol{Q}_\mathrm{l} \end{bmatrix} \boldsymbol{R} \ , \tag{3}$$

i.e., a sorted QR decomposition has to be performed. Thereby, $\boldsymbol{P}$ is an $N_\mathrm{T} \times N_\mathrm{T}$ permutation matrix (a single one in each row and column), $\boldsymbol{Q} = [\boldsymbol{q}_1, \ldots, \boldsymbol{q}_{N_\mathrm{T}}]$ is a $(N_\mathrm{R} + N_\mathrm{T}) \times N_\mathrm{T}$ matrix with orthogonal columns of length $||\boldsymbol{q}_k||$, and $\boldsymbol{R} = [r_{ij}] = [\boldsymbol{r}_1, \ldots, \boldsymbol{r}_{N_\mathrm{T}}]$ is upper triangular with unit main diagonal (decisions in the feedback loop are taken in the order N_T, $N_\mathrm{T} - 1$, ..., 1). The $N_\mathrm{T} \times N_\mathrm{R}$ feedforward matrix for DFE is given by $\boldsymbol{F} = (\boldsymbol{Q}^\mathrm{H}\boldsymbol{Q})^{-1}\boldsymbol{Q}_\mathrm{u}^\mathrm{H} = \mathbf{diag}(||\boldsymbol{q}_1||^{-2}, \ldots, ||\boldsymbol{q}_{N_\mathrm{T}}||^{-2})\boldsymbol{Q}_\mathrm{u}^\mathrm{H}$, and $\boldsymbol{R}$ immediately constitutes the feedback matrix.

Defining a criterion of optimality (usually maximum signal-to-noise ratio in each detection step), the decomposition is unique and can be performed, e.g., via the VBLAST approach [33,13]. One of the most efficient algorithms to perform this task has been propsed by Benesty et al. [3]. Noteworthy, the variances of the noise samples at the decision device, i.e., in the vector $\tilde{\boldsymbol{n}} = \boldsymbol{F}\boldsymbol{n}$, read $\sigma_{\tilde{n},k}^2 = \sigma_n^2 ||\boldsymbol{q}_k||^{-2}$. Hence, for best performance,

$$\min_{k=1,2,\ldots,N_\mathrm{T}} ||\boldsymbol{q}_k||^2 \rightarrow \max \tag{4}$$

should be aspired.

Finally, combining Eqs. (2) and (3), we arrive at the requested factorization task

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{H} \\ \sqrt{\zeta}\boldsymbol{I} \end{bmatrix} \boldsymbol{Z} = \begin{bmatrix} \boldsymbol{Q}_\mathrm{u} \\ \boldsymbol{Q}_\mathrm{l} \end{bmatrix} \boldsymbol{R} \ , \tag{5}$$

where $\boldsymbol{Z} = [z_{ij}] = [\boldsymbol{z}_1, \ldots, \boldsymbol{z}_{N_\mathrm{T}}] = \boldsymbol{T}^{-1}\boldsymbol{P}$ is the combined permutation/change of basis matrix (unimodular Gaussian integer) required at the finial step in the receiver.

The transmission system employing LRA DFE is depicted in Fig. 1.

Noteworthy, for $\boldsymbol{T} = \boldsymbol{I}$, i.e., $\boldsymbol{Z}$ is a pure permutation matrix, conventional, sorted DFE (VBLAST) results.

[2] Notation: $\boldsymbol{A}^\mathrm{T}$: transpose of matrix $\boldsymbol{A}$; $\boldsymbol{A}^\mathrm{H}$: Hermitian (i.e., conjugate) transpose; $\boldsymbol{A}^{-\mathrm{H}}$: inverse of Hermitian transpose; $\boldsymbol{I}$: identity matrix; $\mathbf{diag}(\cdot)$: diagonal matrix with elements taken from the indicated vector/set. $\mathrm{E}\{\cdot\}$: expectation. $||\boldsymbol{x}||$: Euclidean norm of vector $\boldsymbol{x}$. $\lceil x \rfloor$: rounding of real-valued variable x to the next integer. In case of complex-valued x real and imaginary parts are rounded individually, i.e., $\lceil a + \mathrm{j}b \rfloor = \lceil a \rfloor + \mathrm{j}\lceil b \rfloor$.

[3] The set of Gaussian integers is given by $\mathbb{G} = \mathbb{Z} + \mathrm{j}\mathbb{Z}$, i.e., the set of complex numbers with integer real and imaginary part.

[4] For details how to handle the offset caused by drawing a_k from a translate of the Gaussian integers, see, e.g., [30].