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Generalized fuzzy linear programming for decision making under uncertainty: Feasibility of fuzzy solutions and solving approach

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ABSTRACT

In this study, a generalized fuzzy linear programming (GFLP) method is developed for dealing with uncertainties expressed as fuzzy sets. The feasibility of fuzzy solutions of the GFLP problem is investigated. A stepwise interactive algorithm (SIA) based on the idea of design of experiment is then advanced to solve the GFLP problem. This SIA method was implemented through (i) discretizing membership grade of fuzzy parameters into a finite number of α-cut levels, (ii) converting the GFLP model into an interval linear programming (ILP) submodel under every α-cut level, (iii) solving the ILP submodels through an interactive algorithm and obtaining the associated interval solutions, (iv) acquiring the membership functions of fuzzy solutions through statistical regression methods. A simple numerical example is then proposed to illustrate the solution process of the GFLP model through SIA. A comparison between the solutions obtained though SIA and Monte Carlo method is finally conducted to demonstrate the robustness of the SIA method. The results indicate that the membership functions for decision variables and objective function are reasonable and robust.

1. Introduction

Management of environmental problems is a priority for socio-economic sustainable development throughout the world. Amounts of factors should be considered in environmental management, leading to great complexity in actual decision making. Systems analysis techniques have been widely applied to handle above environmental management issues. The systems analysis techniques can be classified into: systems engineering models and systems assessment tools; moreover, the systems engineering models, which involve cost-benefit analysis (CBA), forecasting models (FMs), simulation models (SMs), optimization models (OMs), and integrated modeling system (IMS) can be seen as the core technologies [41]. Among the five models above, OM is a fairly useful tool for supporting effective environmental management [2,49]. Since 1970s, various optimization methods have been applied to environmental management issues, such as air quality management, solid waste management, and water resources management [12–14,19–23,50]. As early as in 1992, Van Beek et al. [49] demonstrated how operational research could play an important role in solving environmental problems.

However, in many real-world environmental management problems, uncertainties exist in various system components and their interrelationships. For example, waste generation rate within a city is related to many socio-economic and

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environmental factors, and exhibits uncertain and dynamic features; the efficiency of a municipal wastewater treatment plant is affected by wastewater flow rate, and is uncertain in nature; regional air quality is mainly influenced by air pollutant emissions within this area, which also present uncertain characteristics [11]. Such uncertainties can lead to increased complexities in the related optimization efforts. Simply ignoring these uncertainties is considered undesired as it may result in inferior or wrong decisions [40,44,51]. Therefore, inexact optimization methods are desired for supporting environmental management under uncertainty.

The paper is organized as follows: in the next section, previous work on inexact optimization methods will be reviewed. In Section 3, a generalized fuzzy linear programming method and the related computational procedures will be introduced and investigated. An illustrative example is given in Section 4. Short conclusions are made in Section 5.

2. Related work

In the past decades, a large number of inexact optimization methods have been developed to deal with various uncertainties in actual management problems. These methods were mainly classified into three categories, namely stochastic, interval and fuzzy mathematical programming (abbreviated as SMP, IMP, and FMP, respectively) [3–5,7,8,11–25,30–38,44]. SMP, derived from probability theory, could deal with various probabilistic uncertainties; however, the increased data requirements for specifying the parameters' probability distributions might affect their practical applicability [39]. IMP, based on interval analysis, was proved to be an effective approach in dealing with uncertainties. IMP did not require distributional information and would not lead to complicated intermediate models. However, it was to be noted that the outputs of IMP were with lower and upper bounds, and thus could not reflect the distribution of uncertainty within the lower and upper bounds [45].

Fuzzy sets theory and the related possibility theory, which were designed to supplement the interpretation of linguistic or measured uncertainties for real-world random phenomena, provided powerful tools for dealing with imprecise information [52]. Although fuzziness offered a weaker (less informative) indication of uncertainty than randomness, fuzzy sets theory and possibility theory did not require sufficient precise data to quantify imprecise information. Furthermore, fuzzy sets were more accurate than interval numbers, and reflected more detailed information for system uncertainties. FMP was widely developed and applied since it was proposed by Tanaka et al. [46] in 1974. Various types of FMP models and related solving approaches were proposed [3,4,7,8,28–30,37,43]. For example, Delgado et al. [7] studied a general model for fuzzy linear programming (FLP) problems which simultaneously involved in the constraints set both fuzzy numbers and fuzzy constraints. Rommelfanger [42,43] presented an interactive method for solving a multi-criteria linear program, where coefficients of the objective functions and/or of the constraints were expressed as LR fuzzy numbers. Jiménez et al. [24] proposed an interactive resolution method for the FLP model with all coefficients were fuzzy numbers. However, most of these methods were just applied to deal with fuzzy parameters or relationships. Few of them focused on solutions expressed as fuzzy sets in FLP problems.

In real-world management problems, fuzzy solutions can provide some distributional information, and thus would be more attractive to decision makers. Tanaka and Asia [47] initially proposed a possibilistic linear programming formulation and applied linear programming technique to obtain the largest possibility distribution of the decision variables. Tanaka et al. [48] then studied several kinds of possibility distributions of fuzzy variables in possibilistic linear programming problems. Buckley and Feuring [3] considered an extreme case of fully fuzzified linear program (FFLP) with all parameters and variables expressed as fuzzy numbers and solved it through an evolutionary algorithm. Hashemi et al. [17] proposed a two-phase approach to find the optimal solutions for the FFLP problem through firstly maximizing the possibilistic mean value of fuzzy objective function and then minimizing the standard deviation of the original fuzzy objective function. Lotfi et al. [35] developed a lexicography method to solve the FFLP problem and generate fuzzy approximate solutions. However, previous studies on fuzzy solutions in FLP problems mainly focused on some special types of fuzzy numbers such as symmetric, triangular, or trapezoidal fuzzy numbers. Furthermore, some of them might also lead to complicated intermediate models and thus were not applicable for large scale problems. Besides, few of previous studies investigated feasibility of fuzzy solutions in FLP problems with all coefficients expressed as fuzzy sets.

Therefore, this study aims to develop a generalized fuzzy linear programming (GFLP) method to reflect ambiguous information in actual management problems. The word "generalized" in GFLP means that the proposed GFLP method is extended from FLP through permitting all parameters to be presented as fuzzy sets, allowing uncertainties to be directly communicated into the optimization process and generating solutions expressed as fuzzy sets. The feasibility of fuzzy solutions for GFLP will be investigated. Then a stepwise interactive algorithm (SIA) will be advanced to solve the GFLP problem. An illustrative example will be proposed to show the solving process of the GFLP problem through SIA. Finally, comparison between solutions obtained though SIA and Monte Carlo method will be conducted to demonstrate the robustness of the SIA method.

3. Methodology

In practical management problems, probabilistic methods are not applicable to quantify various uncertainties when data are insufficient. Consequently, adoption of fuzzy sets theory would be a potential alternative, especially when uncertainties can be consciously assumed by decision makers or experts. In this section, a generalized fuzzy linear programming (GFLP) method will be proposed to deal with uncertainties expressed as fuzzy sets. Meanwhile, the feasibility of fuzzy solutions for GFLP will be illustrated. A stepwise interactive algorithm (SIA) will then be proposed to solve the GFLP model based on some