Economic Modelling 31 (2013) 189–197

Contents lists available at SciVerse ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

# An empirical analysis of the downside risk-return trade-off at daily frequency☆

Benoît Sévi *

Aix-Marseille University (Aix-Marseille School of Economics), CNRS & EHESS, France

ARTICLE INFO

*Article history:*
Accepted 30 November 2012

*Keywords:*
Risk-return tradeoff
Downside-risk
MIDAS regressions
HAR model
Intraday data

ABSTRACT

This paper considers the downside-risk aversion of investors as an explanation for the risk-return trade-off. We test empirically this hypothesis using intraday data along with the recent measure of downside-risk called realized semivariance developed in Barndorff-Nielsen et al. (2010). The empirical analysis over the period 1996–2008 provides evidence of a significant relation between semivariance and excess returns at the daily frequency. To gain better understanding of the relation between returns and downside-risk, we investigate the statistical relation between a new measure of conditional asymmetry, namely the ratio of the downside realized semivariance over the variance, and obtain a revealing pattern using a rolling window framework able to link asymmetry in the distribution to future returns. In particular, the asymmetry measure becomes significant when the past realized variance is not significant any more thereby providing insights about a possible change in the behavior of investors.



## 1. Introduction

The risk-return trade-off is one of the most pressing question in financial economics. Ghysels et al. (2005) even coin it as the "first fundamental law in finance". This is also one of the main puzzles in finance. The puzzle arises from the fact that the ICAPM is not supported empirically. For instance, French et al. (1987) using the data from 1920 to 1984 do not provide evidence of a statistically significant relation between risk and returns. Recently, it has been suggested that the relation may only exist in the (very)-long term (Lundblad, 2007; Bandi and Perron, 2008, etc.). Conversely, Bali and Peng (2006) have shown that it could exist in the (very)-short term, namely at the daily level. In this paper, we examine the predictive power of the semivariance estimator for future returns. We show that this estimator allows to establish a robust relation between (downside-)risk and return in the short run. We thus show that investors' downside-risk aversion may partly explain the empirical puzzle and confirm this finding using a new measure of asymmetry for the distribution of returns.

The problem of the empirical non-recovering of a significant relation for the theoretical ICAPM may have different sources: (i) the inadequacy of the ICAPM itself, (ii) risk preferences of agents who may not only be risk-averse but also exhibit preferences towards higher moments and/or (iii) the inaccuracy of the volatility proxy and the methodology to forecast volatility considered in the literature. In this paper, as in most of the existing literature, we assume the validity of the ICAPM and suggest an empirical approach which deals with the two other issues, by considering both semivariance based on intraday data and a recent method for volatility forecasting. We then test the ICAPM using this nonparametric estimator[1] of conditional semivariance relying on intraday data. Thus, we both consider the downside-risk aversion of investors and the measure of downside-risk itself along with an adequate model of volatility forecasting. We finally combine this measure with the original measure of realized volatility to estimate the asymmetry of the returns' distribution and show that this measure is significant most of the time in our sample. Importantly, it is more significant when the realized volatility is not.

The financial literature has stressed the importance of considering higher order moments in pricing securities (see Dittmar, 2002; Friend and Westerfield, 1980; Lim, 1989 among others). The literature on the relation between the asymmetry in the distribution and expected returns is rooted in the seminal contribution of Arditti (1967) who showed using a Taylor series expansion how to derive the price risk of the third moment. The preference for a positive skewness in returns modifies standard portfolio choice theory when only first and second moments are under consideration. In a series of papers, alternative portfolio choice theories are developed which consider moments beyond variance.[2] This may also modify the equilibrium under consideration as

☆ This paper has been written while the author enjoyed the hospitality of the London Business School. For helpful comments, I would like to thank Sule Alan, Derek Bunn, Victor DeMiguel, Bruce Grundy (EFA discussant), Yannick Le Pen, Fabien Tripier, François-Charles Wolff as well as the participants in the 2010 EFA Annual Meeting in Frankfurt and the seminar participants in Angers and Nantes universities. A referee provided interesting insights that helped to improve significantly the paper. Of course, all errors are my own responsibility.

* GREQAM, DEFI Department (Development and International Finance), Château La Farge – Route des Milles 13290 Les Milles Aix-en-Provence, France
*E-mail address:* benoit.sevi@gmail.com.

[1] The essential of the literature so far has mainly considered parametric models such as GARCH-in-mean (see Glosten et al. (1993) or ; Engle and Ng (1993) among others) from Engle et al. (1987).

[2] See Simkowitz and Beedles (1978), Scott and Horvath (1980), Kane (1982) for earlier contributions and Athayde and Flôres (2004), Harvey et al. (2010), Jarrow and Zhao (2006), Jondeau and Rockinger (2006) or Guidolin and Timmermann (2008) for more recent ones.

0264-9993/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.econmod.2012.11.059

in Kraus and Litzenberger (1976), Friend and Westerfield (1980) or Lim (1989). This leads investors to purchase less diversified portfolio than recommended by mean-variance based theory (see Mitton and Vorkink, 2007). Recently, Chabi-Yo et al. (2008) have derived a four-moment model to extend the two-fund separation theorem of Tobin. The recent contribution by Barberis and Huang (2008) illustrates the trade-off between risk and skewness for investors. In their model, agents have preferences following prospect theory along the lines of Kahneman and Tversky (1979, 1992). Assets with positive skewness are favored by investors who consider "stocks as lotteries" thereby holding nonoptimal portfolios in the Markowitz sense and favoring high-reward/low-probability assets. Brunnermeier et al. (2007) conclude that "The desire for skewness can also impact the market return. If bad aggregate states have low probabilities, as for disasters or peso problems, then it is possible for the desire for skewness to increase the equity premium as investors seek to avoid negative skewness."(p. 164).

Our estimation strategy, which relies on intraday (transaction data) and the estimator developed in Barndorff-Nielsen et al. (2010) (BNKS hereafter), has the strong advantage to use a very robust proxy for the latent volatility as shown in a series of papers since Andersen and Bollerslev (1998). Estimation of volatility using intraday data has only a short but very intense history. Zhou (1996) extended the idea by French et al. (1987) following Merton's (1980) argument to build the first estimator. Barndorff-Nielsen and Shephard (2002) gave theoretical foundations to this literature. The main idea is to use the theory of quadratic variation which states that the sum of squared intraday returns delivers an estimate of the current variance of a stochastic process which is more and more precise the finer is the partition used to compute the intraday returns (infill asymptotics). BNKS (2010) use the same idea but only consider signed (negative or positive) intraday returns to estimate the downside realized semivariance. In words, the concept of downside semivariance is related to the variability of a given process when only signed changes of this process are considered, i.e. when the process exhibits downside moves.

These "realized" estimators of volatility are now widely used in various economic contexts (see Tauchen and Zhou, 2011; or Wright and Zhou, 2009 among many others) and for various financial markets (see for instance Liu and Wan, 2012 who investigate the empirical properties of price volatility in the Shanghai fuel oil futures). Another way to estimate volatility is naturally by using option prices. A number of recent studies have investigated the relation between returns and skewness using model-free implied skewness or asymmetry either at the cross-sectional level (Conrad et al., forthcoming; Cremers and Weinbaum, 2010; Rehman and Vilkov, 2012; Xing et al., 2010) or, as we do, at the aggregate level (Atilgan et al., 2010; Santa-Clara and Yan, 2010).[3] In these papers, the relative price of put options vis-à-vis call options is shown to provide some information about future returns.[4] The so-called "smirk" (difference between the out-of-the-money put and the at-the-money call implied volatility), a measure of *ex ante* asymmetry in the return distribution, is agood predictor of future returns thereby proving the relevance of risk-neutral measure for the risk-return debate. The important role of VIX in estimating a risk-return relationship has recently been highlighted in Kanas (2012).

Our empirical results for the S&P 500 index futures over the period 1996–2008 indicate that downside-risk aversion is important in predicting future returns, at least as much as risk aversion. In particular, in the same vein as Patton and Sheppard (2011), we obtain comparable estimates for the realized variance and for the downside realized semivariance. This has implications for our understanding of the risk-return tradeoff as well as for the significance of the various components of realized variance – negative and positive realized semivariances being two of them.[5] To further test the downside-risk aversion, we use a new measure of conditional asymmetry derived from realized semivariance and variance and show that this measure help in recovering the risk-return tradeoff in recent years. The rest of the paper is organized as follows. In the next section, we present shortly the estimators that we use along with the intraday dataset. In Section 3, empirical results are presented for the different specifications using the full sample and then rolling windows. Section 4 proposes the new measure for asymmetry in the distribution of returns and provides an estimation of its significance in a ICAPM-like regression. Section 5 concludes.

## 2. Some econometrics of high-frequency data

Let us assume that $p(t)$, the logarithm of the asset price, follows the general stochastic volatility jump diffusion model:

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) + \kappa(t)dJ(t) \quad with \quad t \geq 0 \tag{1}$$

where the mean $\mu(.)$ (predictable drift) is assumed to be continuous and locally bounded, and the instantaneous (spot) volatility $\sigma(.)$ is strictly positive and càdlàg (right continuous with left limit). The mean, as well as the spot volatility, are both assumed to be independent from the driving standard Brownian motion $W(.)$.[6] The finite activity counting process, noted $J(t)$, is normalized such that $dJ(t)=1$ when a jump occurs at time $t$, and $dJ(t)=0$ otherwise. Finally, $\kappa(t)$ is the jump size at time $t$, which is assumed to be random. The process in Eq. (1), which belongs to the class of Brownian semimartingale processes with jumps, allows returns to exhibit leptokurticity and volatility clustering, which are both relevant empirical characteristics for financial time-series.

The theory of quadratic variation allows to derive nonparametric volatility measures, and thus to decompose the total price variation into its continuous and jump components. If we define $[p](t)$ as the quadratic variation of the process in Eq. (1), then:

$$[p](t) = \text{plim} \sum_{j=0}^{n-1} \left[ p\left(\tau_{j+1} - p\left(\tau_j\right)\right) \right]^2 \tag{2}$$

where $0=\tau_0<\tau_1<\ldots<\tau_n=t$ is a sequence of partitions, and $sup_j \{\tau_{j+1}-\tau_j\} \to 0$ for $n \to \infty$. The quadratic variation of process in Eq. (1) may then be expressed as:

$$[p](t) = \int_0^t \sigma^2(\tau)d\tau + \sum_{j=1}^{J(t)} \kappa^2\left(t_j\right) \tag{3}$$

where $t_j$ are times when a jump occurs, implying that $dJ(t_j)=1$. This decomposition allows to identify the integrated volatility, which is due to the continuous sample path movement of the process, and the jump component, which is the sum of squared jumps. Eq. (3)

[3] Atilgan et al. (2010) use two different measures of skewness: one is extracted from option prices (risk-neutral) while the other one is computed by using historical data (physical measure). Conrad et al. (forthcoming) also rely on a risk-neutral measure. Ang et al. (2006) provide evidence of a downside risk premium using a downside risk measure related to the CAPM (positive and negative betas). In the present paper we also rely on historical data and show that these data do convey some information for the prediction of future returns.

[4] In the case of Cremers and Weinbaum (2010), the liquidity of both the option and the stock also plays a role in explaining future returns. Namely, "The degree of predictability is larger when option liquidity is high and stock liquidity is low, while there is little predictability when the opposite is true."

[5] Alternatively, the jump component may also be considered in future research to show its explanatory power in the risk-return relationship. This is of course a different issue that we do not investigate in the present paper.

[6] The independence assumption is discussed in length in Barndorff-Nielsen and Shephard (2006), who explain that it rules out leverage and volatility feedback effects. The absence of leverage has been recently shown to be relevant empirically for S&P 500 index and futures returns (see Bollerslev et al., 2006, 2009a among others).