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Elastoplastic analysis of frames composed of softening materials by restricted basis linear programming

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ABSTRACT

This paper is concerned with nonlinear analysis of frames composed of softening materials. The previously proposed dissipated energy maximization approach is extended to determine non-holonomic solution of such frames. The adopted assumptions are: linear kinematics, lumped plasticity with softening behavior, piecewise-linear yield functions, associate flow rule and isotropic evolution with a three phase linear softening rule. The approach is based on a mathematical programming formulation. The solution procedure is discussed and presented in a comprehensive flowchart. It is shown that this method has the ability of solving and tracing path dependent problems and detecting any possible bifurcation.

1. Introduction

Elasto-plastic analysis of frames with softening material has been the subject of research and interest for many researchers during last decades (see e.g., [1,2]). The necessity of more realistic solutions for frames subject to increasing lateral loads, makes this topic a well-known and still growing area among other engineering research topics [3–5]. To this end some simplifying assumptions are found to be practical and efficient. Among them are: lumping plasticity in some pre-selected sections, using piecewise-linear yield surfaces and ignoring path dependency and possible local unloading (i.e., "holonomic" behavior) over each load step. Beside these simplifying assumptions, optimization is found to be a very powerful and versatile computational tool for the direct analysis of nonlinear problems [6]. A brief review of such approaches in the field follows.

Linear Programming (LP) has long been recognized as a suitable tool for limit state analysis of structures. In the classical limit analysis, when loads are applied to a framed structure and are assumed to increase proportionally, on the basis of the well-known lower- and upper-bound theorems the limit load and the collapse configuration can be computed as a solution to a linear mathematical programming problem [7,8]. This approach is known to be a milestone in the history of structural mechanics and still is a common approach for engineers in a variety of practical problems, but also a developing subject in the literature (see [2,9]).

Maier [10,11] proposed the use of mathematical programming in elasto-plastic analysis of structures, for which the nonlinear holonomic response is sought as solution to a Quadratic Programming (QP) problem. Later on, the method was extended to consider the interaction of axial force and bending moment by adopting a piece-wise linear yield surface [12]. In order to increase the efficiency of the solution procedure, in some later researches the QP formulation was replaced by a Linear Complementarity Problem (LCP), [13], and by a Restricted Basis Linear Programming (RBLP) [14]. This approach was also improved and generalized to shakedown and nonlinear dynamic analyses [15,16]. Cocchetti and Maier implemented the aforementioned approach in the analysis of softening frames [17]. They proposed two procedures, namely a step-by-step method (SBSM) and a stepwise holonomic/fully holonomic analysis, and discussed them in detail. In both solution schemes, the load multiplier is considered as the objective function to be maximized and the structural response is sought as the solution(s) to an LCP. Load factor maximization has also been employed by Lógó and Taylor in developing the so called "Extremum Principle" [18] and implemented by Kaliszky and Lógó in analysis of truss structures [19]. Clearly, in the case of softening negative load increments are expected and, as a consequence, the load maximization principle fails. This problem was discussed in [17], and in such a case, a new solution to LCP for negative load increments was proposed as a remedy. Another holonomic approach was proposed by Tangaramvong and Tin-Loi to deal with structures governed by piecewise linear softening models [20]. This method employs a penalty approach to solve the nonlinear optimization problem, but the penalty parameter that enforces complementar-

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ity has to be carefully selected to prevent numerical instabilities. Some recommendations are given on the selection of the penalty coefficient, but it seems that this parameter and its updating schedule do not follow any clear rule.

Recently, Mahini et al. [21] proposed a new dissipated energy maximization (DEM) approach in which the solution to the incremental LCP (in terms of plastic multipliers) is obtained as the solution to an LP problem by exploiting a plastic work criterion. This approach has shown to have distinct ability in solving frames composed of elastic-perfectly-plastic materials. In this approach, the incremental LP problem is formulated in terms of all problem variables (i.e., load and plastic multipliers), just like it is done in other holonomic formulations; but as a result of additional restrictions applied to the vector of basic variables, "exactness" and "stability" features of a step-by-step method are preserved. This approach is characterized by a non-holonomic nature and it can perform any required local unloading: the resulting refreshed Simplex table always represents the updated configuration and, in this sense, the loading process can continue without any extra computational effort.

However the dissipated power has been frequently employed as an internal variable for hardening/softening constitutive model construction, (see e.g., [22]), in DEM approach maximization of the dissipated power is used as a tool for recognizing the correct path of plastic deformations in the space of plastic multipliers.

In this paper, the DEM solution strategy is extended to the softening frames and is shown to be capable of tracing the exact response by detecting any elastic unloading and any equilibrium bifurcation. For this purpose, the theoretical aspects of the method are discussed in Section 2. The piecewise linear softening model, with interacting planes, for a typical frame section (or "joint") is developed in Section 2.1. The problem formulation is illustrated in Section 2.2. Some terms and definitions regarding the solution algorithm are given in Section 2.3. The solution to the incremental LP problem is described in Section 3. Afterwards, some numerical examples are discussed in Section 4 to demonstrate the robustness of the proposed algorithm and show its capabilities. Finally, Section 5 is devoted to discussions and final conclusions.

2. Theoretical aspects

As stated above, the current version of DEM has been proposed for frames with elastic-perfectly-plastic behavior. In order to modify the algorithm for the softening frame analyses, the formulation is updated and some modifications are discussed in the following.

2.1. Piecewise-linear softening model for the joint

Piecewise-linearized constitutive models are reasonable assumptions and effective formulation tools in nonlinear analysis of framed structures (see e.g., [23,24]). In this study, a 6-mode piecewise-linear yield surface is considered and an isotropic softening rule is adopted to describe the evolution of the yield locus (see Fig. 1a).

For the joint s, a PWL yield surface with isotropic softening can be given the following mathematical description:

$$Y_s = \boldsymbol{\Phi}_s^{\mathrm{T}} \mathrm{f}_s - \alpha_s \leqslant 0, \tag{1}$$

where, f_s is the vector of generalized stresses (axial force, N_s, and bending moment, M_s) acting on the cross-section s:

$$\mathrm{f}_s = \begin{Bmatrix} N_s \\ M_s \end{Bmatrix}. \tag{2}$$

Matrix $\boldsymbol{\Phi}_s$ contains the normal vectors of the yield planes (normalized w.r.t. the corresponding plastic capacities N_p and M_p); for the 6-mode PWL yield locus shown in Fig. 1a, ($m = 6$) it assumes the following form:

$$\boldsymbol{\Phi}_s = \left[\begin{array}{c|c|c|c|c|c} \frac{\varphi^s_{N1}}{N_p} & \frac{\varphi^s_{N2}}{N_p} & \frac{\varphi^s_{N3}}{N_p} & \frac{\varphi^s_{N4}}{N_p} & \frac{\varphi^s_{N5}}{N_p} & \frac{\varphi^s_{N6}}{N_p} \\ \frac{\varphi^s_{M1}}{M_p} & \frac{\varphi^s_{M2}}{M_p} & \frac{\varphi^s_{M3}}{M_p} & \frac{\varphi^s_{M4}}{M_p} & \frac{\varphi^s_{M5}}{M_p} & \frac{\varphi^s_{M6}}{M_p} \end{array}\right]_{2\times m}. \tag{3}$$

In the above formula, indices 1–6 refer to the yield planes and ϕ_{ij} is the partial derivative of the jth yield plane with respect to ith generalized stress.

Each entry α_j^s of vector α_s is related to the distance of the jth yield plane from the origin, so that α_s determines the size/shape of yield surface at any loading instance. According to the isotropic softening rule, it is convenient to relate the size change of the yield locus to an internal scalar variable describing the irreversible evolution. To this end, a generalized equivalent plastic strain, ε_p^s, is defined as a linear combination of plastic multipliers, x_s (which is a $m \times 1$ vector and for the selected 6-mode piecewise-linear yield surface $m = 6$), of the joint model as follows:

$$\varepsilon_\mathrm{p}^s = \overline{\mathbf{Q}}_s \mathrm{x}_s. \tag{4}$$

$\overline{\mathbf{Q}}_\mathbf{s}$ is a row-vector collecting the magnitudes of the yield locus normal vectors:

$$\overline{\mathbf{Q}}_\mathbf{s} = [\, n_s^1 \quad n_s^2 \quad n_s^3 \quad n_s^4 \quad n_s^5 \quad n_s^6 \,]_{1\times m}. \tag{5}$$

Plastic behavior of softening frame joints is usually simplified in three general phases: initial yielding phase, softening phase and ultimate strength phase. In the first and last phases, the material is assumed to behave fully plastic; instead, in the second phase the strength decreases due to softening. Fig. 1c shows such a 3-phase softening rule. Parameters ε_{s1}^p and ε_{s2}^p are the equivalent plastic strain limits at which the softening branch and final limit phase begin, respectively. The evolution law depicted in Fig. 1c can be mathematically described by three (non-negative) plastic multipliers ($\bar{x}_s^0, \bar{x}_s^1$ and $\bar{x}_s^2$) and the following relationships:

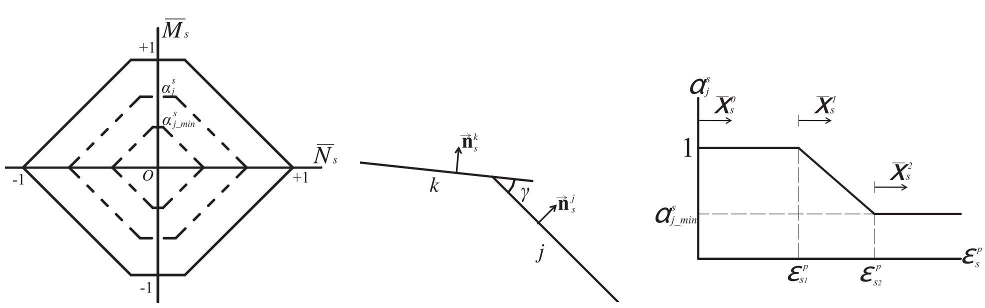

Fig. 1. Constitutive softening model in terms of normalized internal actions: (a) typical 6-mode piecewise linear yield surface and its evolution with isotropic softening, (b) intersection of two adjacent yield planes (j and k) and their corresponding normal vectors, and (c) three phase softening rule.