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# Sensitivity analysis of Values at Risk

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**Abstract**

The aim of this paper is to analyze the sensitivity of Value at Risk (VaR) with respect to portfolio allocation. We derive analytical expressions for the first and second derivatives of the VaR, and explain how they can be used to simplify statistical inference and to perform a local analysis of the VaR. An empirical illustration of such an analysis is given for a portfolio of French stocks. 

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*Keywords:* Value at Risk; Risk management; VaR efficient portfolio; isoVar; Kernel estimators; Quantile

## 1. Introduction

Value at Risk (VaR) has become a key tool for risk management of financial institutions. The regulatory environment and the need for controlling risk in the financial community have provided incentives for banks to develop proprietary risk measurement models. Among other advantages, VaR provide quantitative and synthetic measures of risk, that allow to take into account various kinds of cross-dependence between asset returns, fat-tail and non-normality effects, arising from the presence of financial options or default risk, for example.

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There is also growing interest on the economic foundations of VaR. For a long time, economists have considered empirical behavioural models of banks or insurance companies, where these institutions maximise some utility criteria under a solvency constraint of VaR type (see Gollier et al., 1996; Santomero and Babbel, 1996 and the references therein). Similarly, other researchers have studied optimal portfolio selection under limited downside risk as an alternative to traditional mean-variance efficient frontiers (see Roy, 1952; Levy and Sarnat, 1972; Arzac and Bawa, 1977; Jansen et al., 1998). Finally, internal use of VaR by financial institutions has been addressed in a delegated risk management framework in order to mitigate agency problems (Kimball, 1997; Froot and Stein, 1998; Stoughton and Zechner, 1999). Indeed, risk management practitioners determine VaR levels for every business unit and perform incremental VaR computations for management of risk limits within trading books. Since the number of such subportfolios is usually quite large, this involves huge calculations that preclude online risk management. One of the aims of this paper is to derive the sensitivity of VaR with respect to a modification of the portfolio allocation. Such a sensitivity has already been derived under a Gaussian and zero mean assumption by Garman (1996, 1997).

Despite the intensive use of VaR, there is a limited literature dealing with the theoretical properties of these risk measures and their consequences on risk management. Following an axiomatic approach, Artzner et al. (1996, 1997) (see also Albanese, 1997 for alternative axioms) have proved that VaR lacks the subadditivity property for some distributions of asset returns. This may induce an incentive to disagregate the portfolios in order to circumvent VaR constraints. Similarly, VaR is not necessarily convex in the portfolio allocation, which may lead to difficulties when computing optimal portfolios under VaR constraints. Beside global properties of risk measures, it is thus also important to study their local second-order behavior.

Apart from the previous economic issues, it is also interesting to discuss the estimation of the risk measure, which is related to quantile estimation and tail analysis. Fully parametric approaches are widely used by practitioners (see, e.g. JP Morgan Riskmetrics documentation), and most often based on the assumption of joint normality of asset (or factor) returns. These parametric approaches are rather stringent. They generally imply misspecification of the tails and VaR underestimation. Fully non-parametric approaches have also been proposed and consist in determining the empirical quantile (the historical VaR) or a smoothed version of it (Harrel and Davis, 1982; Falk, 1984, 1985; Jorion, 1996; Ridder, 1997). Recently, semi-parametric approaches have been developed. They are based on either extreme value approximation for the tails (Bassi et al., 1997; Embrechts et al., 1998), or local likelihood methods (Gouriéroux and Jasiak, 1999a).

However, up to now the statistical literature has focused on the estimation of VaR levels, while, in a number of cases, the knowledge of partial derivatives of VaR with respect to portfolio allocation is more useful. For instance, partial