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A differential equation approach to fuzzy vector optimization problems and sensitivity analysis

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Abstract

The first objective of the analysis presented in this paper is to extend the technique while using differential equations approach for solving fuzzy non-linear programming problem in solving vector optimization problems with fuzzy parameters (VOP-FP). This technique is based mainly on using differential equations approach, which is very effective in finding many local α-Pareto optimal solutions, where the (VOP-FP) is transformed to fuzzy nonlinear autonomous system of differential equations and the relation between the critical points of differential system and local α-Pareto optimal solutions of original optimization problem is proved.

The second objective of the analysis presented in this paper is to obtain sensitivity information for (VOP-FP) by using the technique of trajectory continuation.

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1. Introduction

A great deal of work has been done in the field of fuzzy vector optimization problems and sensitivity analysis. From the recent work in this direction, let us mention [2–14]. In [12] Sakawa and Yano presented an interactive decision-making method for multi-objective nonlinear programming problems with fuzzy parameters. In that work, the fuzzy parameters have been characterized by fuzzy numbers and the concept of α-Pareto optimality has been introduced. From the earlier work in this direction by using differential equation approach [10], we presented a technique for solving multi-objective nonlinear programming problems [5], then we extended this result for solving fuzzy nonlinear programming problems [6]. Also, we presented sensitivity analysis for parametric vector optimization problems [7]. In this paper, we shall be concerned with differential equation approach for solving vector optimization problems with fuzzy parameters (VOP-FP) and sensitivity information. This approach is very effective in finding many local α-Pareto optimal solutions.

The paper is organized as follows. In Section 2 we formulate the fuzzy vector optimization problem involving fuzzy parameters in the objective functions. In Section 3, the nonlinear autonomous differential system (Fundamental Equations) for solving (VOP-FP) is introduced. Also, the relation between the critical points

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of the differential system which is asymptotically stable and local α-Pareto optimal solutions of the original (VOP-FP) is presented. Finally, in Section 4 a general formula for sensitivity information is also presented. This work is based mainly on the idea of autonomous system of differential equations, by using the technique of trajectory continuation [9,11,14].

2. Problem formulation

In this paper, we consider the following vector optimization problems involving fuzzy parameters in the objective functions in the form:

$$\begin{aligned}\text{(VOP-FP):}\quad \min\quad & (f_1(X,\boldsymbol{\lambda}_1), f_2(X,\boldsymbol{\lambda}_2),\dots, f_m(X,\boldsymbol{\lambda}_m))\\ \text{s.t.}\quad & X\in M=\{X\in R^n:\ G(X)\leqslant 0\},\\ & G=(g_1,g_2,\dots,g_r)\end{aligned} \tag{2.1}$$

where $\Re^n$ is an n-dimensional Euclidean space, $f_i(x,\boldsymbol{\lambda}_i)$, $i=1,2,\dots,m$ and $g_j(X)$, $j=1,2,\dots,r$ possess continuous partial derivatives, where $\boldsymbol{\lambda}_i=(\boldsymbol{\lambda}_{i1},\boldsymbol{\lambda}_{i2},\dots,\boldsymbol{\lambda}_{ip_i})$ fuzzy parameters involved in the objective functions. Here the fuzzy parameters are assumed to be characterized by fuzzy numbers as introduced in [2,3,12], for this, a membership function $\mu_{\boldsymbol{\lambda}_{il}}(\lambda)$, $(i=1,2,\dots,m,\ l=1,2,\dots,p_i)$ is defined for a real fuzzy number $\boldsymbol{\lambda}_{ip_i}$, where $\boldsymbol{\lambda}_{ip_i}$ is a convex continuous fuzzy subset of the real line [12]. For simplicity in the notation, we define the following vectors:

$$\boldsymbol{\lambda}=(\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2,\dots,\boldsymbol{\lambda}_p),\quad \lambda=(\lambda_1,\lambda_2,\dots,\lambda_p),\quad p\leqslant n.$$

Definition 2.1. The α-level set of as the number $\boldsymbol{\lambda}_{il}$, $(i=1,2,\dots,m,\ l=1,2,\dots,p_i)$ is defined ordinary set $L_\alpha(\boldsymbol{\lambda})$ for which the degree of their membership functions exceed the level α; $L_\alpha(\boldsymbol{\lambda})=\{\lambda:\ \mu_{\boldsymbol{\lambda}_{il}}(\lambda_{il})\geqslant\alpha\ (i=1,2,\dots,m,\ l=1,2,\dots,p_i)\}$. For a certain degree α, problem (1) can be formulated as the following non-fuzzy α-VOP, where

$$\begin{aligned}(\alpha\text{-VOP}):\quad \min\quad & (f_1(X,\lambda_1), f_2(X,\lambda_2),\dots, f_m(X,\lambda_m))\\ & X\in M,\\ & \lambda\in L_\alpha(\boldsymbol{\lambda}),\\ \text{s.t.}\quad & M=\{X\in\Re^n:\ g_j(X)\leqslant 0,\quad j=1,2\dots,r\},\\ & L_\alpha(\boldsymbol{\lambda})=\{\lambda:\ \mu_{\boldsymbol{\lambda}_{il}}(\lambda)\geqslant\alpha,\quad i=1,2,\dots,m,\ l=1,2,\dots,p_i\}.\end{aligned} \tag{2.2}$$

Definition 2.2. $X^*\in M$ is said to be α-Pareto optimal solution of (α-VOP) iff there does not exist another $(X,\lambda)\in M\times L_\alpha(\boldsymbol{\lambda})$ such that $f_i(X,\lambda_i)\leqslant f_i(X^*,\lambda_i^*)$, $i=1,2,\dots,m$ with strict inequality holding for at least one i, where the corresponding values of parameter λ^* are called α-level optimal parameters and × denotes the Cartesian product.

Definition 2.3. $X^*\in M$ is said to be α-Pareto optimal solution of (α-VOP) iff there does not exist another $(X,\lambda)\in M\times L_\alpha(\boldsymbol{\lambda})\cap N(X^*,\lambda^*;r)$ such that $f_i(X,\lambda_i)\leqslant f_i(X^*,\lambda_i^*)$, $i=1,2,\dots,m$ with strict inequality holding for at least one i, where the corresponding values of parameter λ^* are called α-level optimal locally parameters and $N(X^*,\lambda^*;r)$ denotes the neighbourhood of (X^*,λ^*) with radius r.