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Structural parameter sensitivity analysis of cantilever- and bridge-type accelerometers

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Abstract

The paper deals with the analytical estimation of bandwidth and device sensitivity of cantilever- and bridge-type monolithic piezoresistive and capacitive accelerometers using different beam models and of their design sensitivity due to the change of geometrical parameters. Using a simplified two-beams model, closed-form formulae have been derived to give the relationship between the rates of length, width and thickness of the beams and the lowest eigenfrequency characterizing the bandwidth as well as the physical sensitivity of the accelerometers. By means of symbolic derivation, a structural design sensitivity analysis has been carried out to obtain information for the optimal selection of the geometrical parameters.

Keywords: Piezoresistive and capacitive sensors; Accelerometers; Frequency analysis; Sensitivity analysis

1. Introduction

Silicon-based accelerometers are typical monolithic sensors [1]. Its simplest form consists of a thin cantilever carrying a large seismic mass (Fig. 1). The view of the cross-section shows the cantilever fixed on its left end. The right end is free — the edge of the frame (as thin line) is beyond the cantilever. It is also possible to suspend the mass from both sides as a bridge (Fig. 2). Both configurations can be used for measuring acceleration.

In piezoresistive sensors, the piezoresistive element is formed in the cantilever or in the bridge. The bandwidth of an accelerometer is given by its first (lowest) natural frequency. The higher is the first natural frequency, the highest signal frequency can be measured. Damping effects are neglected. In real elastic structures, damping slightly diminishes the natural frequency and hinders an infinite amplitude in resonance cases. The device sensitivity is defined as the relative change of resistance per unit of acceleration g

$$S = \frac{\Delta R}{Rg}, \tag{1}$$

where $\Delta R/R = K\varepsilon$. The factor K depends on the orientation of the piezoresistor relative to the crystal and its doping level, while the strain ε is linear function of the stresses. In most cases of MEMS, the main contribution is the normal direction, therefore ε can be approximated by the σ_{m}, normal stress in the place of the piezoresistive element

$$\varepsilon = \frac{\sigma_{\mathrm{m}}}{E}, \tag{2}$$

where E is the Young's-modulus of the material. Combining Eqs. (1) and (2), and introducing $K_{\mathrm{p}} = K/E$, the physical sensitivity of the device is

$$S = K_{\mathrm{p}}\sigma_{\mathrm{m}}, \tag{3}$$

which means that the device sensitivity of a piezoresistive accelerometer is proportional to the normal stress in the place of the piezoresistor.

In capacitive accelerometers, one-plate of the capacitor is stationary, i.e. connected to the housing and the other plate is attached to the seismic mass. Due to acceleration, the mass deflects, consequently changing the distance between the two-plates. In analogy to the piezoresistive sensor, the sensitivity S is defined as the relative change of capacitance per unit of acceleration

$$S = \frac{\Delta C}{Cg}, \tag{4}$$

where $C = \varepsilon_0 A/d_0$ is the rest capacitance (when no input signal is applied) and $\Delta C = C' - C$, a nonlinear function, since the capacitance C' is inversely proportional to the

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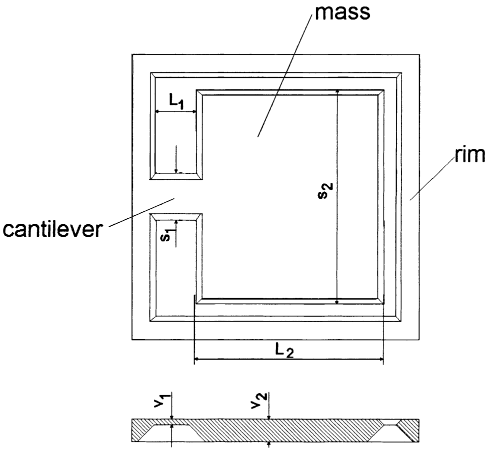

Fig. 1. Cantilever-type accelerometer.

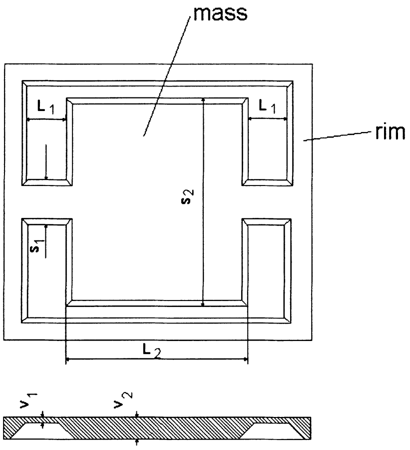

Fig. 2. Bridge-type accelerometer.

distance between the electrodes. For bridge-type sensors

$$C' = \frac{\varepsilon_0 L_2 s_2}{d_0 - z_\mathrm{m}}, \tag{5}$$

where ε_0 is the free-space permittivity, d_0 the gap at rest between the plates, and z_m the vertical deflection of the middle of the movable capacitor plate. Thus, using $\Delta d = d_0 - z_\mathrm{m}$, the device sensitivity of a capacitive sensor is

$$S = \frac{(d_0/\Delta d) - 1}{g}. \tag{6}$$

The aim of structural design in several applications is to achieve large bandwidth and high sensitivity of the sensor. A correctly designed accelerometer should have large bandwidth ensuring a large range of flat frequency response [2].

2. Simplified "beam and mass" models

Neglecting the slopes of the seismic mass due to anisotropic etching and considering the supporting rim as a built-in end, we obtain a cantilever or a bridge consisting of two-elastic beams of different rectangular cross-sections (Fig. 3). Since, the thickness of the seismic mass is in general much larger than that of the supporting beam, this two-elastic-beams model can be simplified without much loss of accuracy. The simplest model is a one degree-of-freedom vibrating system, if the seismic mass is considered as a particle [3]. The natural frequency is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{3I_1E}{m_2(L_1 + L_2/2)^3}} \tag{7}$$

for the cantilever and

$$f = \frac{1}{2\pi}\sqrt{\frac{24I_1E}{m_2L_1^3}} \tag{8}$$

for the bridge. Here, $m_2 = \rho L_2 s_2 v_2$ is the seismic mass and $I_1 = s_1 v_1^3/12$ the moment of inertia of the elastic beam. In this case, the mass of the supporting beam is neglected, therefore the maximum normal stress due to bending is

$$\sigma_\mathrm{m} = \frac{m_2 g(L_1 + L_2/2)v_1}{2I_1} \tag{9}$$

for the cantilever and

$$\sigma_\mathrm{m} = \frac{m_2 g L_1 v_1}{4I_1} \tag{10}$$

for the bridge. The maximum deflections are as follows:

cantilever

$$z_\mathrm{m} = \frac{m_2 g(L_1 + L_2/2)^3}{3I_1E} \tag{11}$$

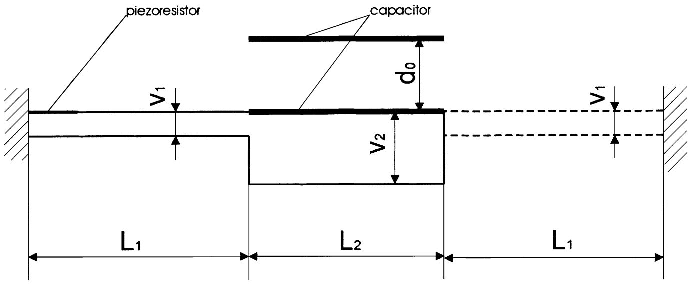

Fig. 3. Simplified two-beams model (dashed line refers to the bridge-type sensor).