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Robust PSS design by probabilistic eigenvalue sensitivity analysis

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Abstract

When a wide range of system operation is taken into account for power system dynamic studies, probabilistic eigenvalue analysis efficiently provides the statistical distributions of concerned eigenvalues. Under the assumption of normal distribution, each eigenvalue can be described by its expectation and variance. To enhance system damping under multi-operating conditions by power system stabilizers (PSSs), effects of PSSs on both eigenvalue expectation and variance should be investigated. In this paper, the conventional eigenvalue sensitivity analysis has been extended to probabilistic environment. Eigenvalue sensitivities for both expectation and variance are determined to form two types of probabilistic sensitivity indices (PSIs). Robust PSS locations are selected by one type of PSI, PSS parameters are tuned by the probabilistic sensitivity analysis using another type of PSI.

Keywords: Dynamic stability; Power system stabilizer (PSS); Eigenvalue; Sensitivity; Probability

1. Introduction

To improve system dynamic damping by PSSs, many indices and techniques have been proposed for PSS site selection and parameter optimization. A comparative study was presented in [1] and the most popular indices for PSS locations were identified as the residue method [2] and the damping torque analysis. Relationships among different indices were discussed under special conditions and the computation precision was also compared [1]. Modal analysis [3], damping torque approach [4] and the eigenvalue sensitivity analysis [3–5] have been commonly employed for PSS design. A coordinated PSS design approach was presented based on the reduced characteristic equation [6], and the PSS parameters can be 'directly' calculated from the desired eigenvalue assignments. This kind of 'direct' approach was also employed in [7,8].

However, these indices or techniques [1–8] can be regarded as deterministic approaches with constant system parameters and a particular load level. If different operating conditions are considered, the same procedure has to be executed repeatedly and the computing time rapidly increases. Variations in parameters and system operating conditions can be treated by the probabilistic such that the algorithm complexity and computation requirement are independent of the selected sample number.

The probabilistic approach was firstly used for power system dynamic studies in 1978 [9]. The probabilistic property of an eigenvalue was determined from the known statistical attributes of system parameters, such as the rotor angle and mechanical damping. Based on operating curves of nodal injections, multi-operating conditions of a power system were considered in [10]. With nodal voltages determined by stochastic load flow calculation, the probabilistic distribution of each eigenvalue was obtained from the probabilistic attributes of nodal voltages. Under normal distribution, the random property of an eigenvalue is described by its expectation and variance.

Considering multi-operating conditions in this paper, the probabilistic approach is applied to robust PSS design. Taking account the statistical nature of eigenvalues, two types of extended probabilistic sensitivity indices are developed for PSS site selection and parameter adjustment respectively. Initial values of PSS gains and time constants are determined by probabilistic eigenvalue sensitivity analysis. All PSS parameters are tuned by using a PSI matrix.

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2. Probabilistic eigenvalue analysis

Under multi-operating environment, nodal powers, nodal voltages and system eigenvalues are all regarded as random variables and expressed by their expectations and variances with the assumption of normal distribution. In this study, system operating samples are created from different standardized daily operating curves of nodal powers and PV voltages, from which expectations and covariances of nodal injections are obtained. By means of the probabilistic load flow calculation, expectations and covariances of nodal voltages are computed.

To determine the probabilistic attributes of eigenvalues, a particular complex eigenvalue λ_k can be analytically expressed as an nonlinear function of the nodal voltage vector $\mathbf{V}$ as:

$$\lambda_k = G_k(\mathbf{V}) \quad (1)$$

In a power system of N nodes, the voltage vector contains $2N$ real components as: $\mathbf{V} = [V_1, V_2, ..., V_{2N}]^T$. As stated in [10,11], eigenvalue expectations are obtained from voltage expectations, which is similar to common deterministic eigenvalue analysis. From the linearised expression of (1) as:

$$\lambda_k = \bar{\lambda}_k + \sum_{i=1}^{2N} \left(\frac{\partial \lambda_k}{\partial V_i}\bigg|_{V=\bar{V}} \Delta V_i \right) \quad (2)$$

the covariance between eigenvalues λ_m and λ_n is derived by:

$$C_{\lambda_{m,n}} = E[(\lambda_m - \bar{\lambda}_m)(\lambda_n - \bar{\lambda}_n)] = \sum_{i=1, j=1}^{2N,2N} \left(\frac{\partial \lambda_k}{\partial V_i} \frac{\partial \lambda_k}{\partial V_j} C_{V_{i,j}} \right) \quad (3)$$

In (2) and (3), the expectation operator is expressed as $\overline{(.)}$ or $E(.)$. $C_{V_{i,j}}$ stands for the nodal voltage covariance between V_i and V_j, i.e. $C_{V_{i,j}} = \overline{\Delta V_i \Delta V_j}$. (3) can also be written in matrix form as (4) with $\mathbf{J}_\lambda$ denoting the first order eigenvalue sensitivity matrix:

$$\mathbf{C}_\lambda = \mathbf{J}_\lambda \mathbf{C}_V \mathbf{J}_\lambda^T \quad (4)$$

Both $\mathbf{C}_\lambda$ and $\mathbf{C}_V$ are symmetrical matrices. Diagonal elements of matrix $\mathbf{C}_\lambda$ are the variances of eigenvalues, and the off-diagonal elements are covariances between eigenvalues. Therefore, the covariances of eigenvalues are determined from voltage covariances using the first order eigenvalue derivatives.

For a particular eigenvalue $\lambda_k = \alpha_k + j\beta_k$, the real part α_k with expectation $\bar{\alpha}_k$ and standard deviation σ_{α_k} (square root of variance) will distribute within $\{\bar{\alpha}_k - 4\sigma_{\alpha_k}, \bar{\alpha}_k + 4\sigma_{\alpha_k}\}$ with the probability 0.99993 that is very close to unity. The acceptable coefficient value of σ_{α_k} can be selected from the range of 3 to 4 [10,13], 4 is used in this paper. To ensure the stability of α_k, its distribution range $\{\bar{\alpha}_k - 4\sigma_{\alpha_k}, \bar{\alpha}_k + 4\sigma_{\alpha_k}\}$ should be located on the left-hand side on the complex plane, which can also be represented by the upper limit α'_k or a standardized expectation α^*_k as:

$$\alpha'_k = \bar{\alpha}_k + 4\sigma_{\alpha_k} \le 0 \quad (5a)$$

$$\alpha^*_k = -\bar{\alpha}_k / \sigma_{\alpha_k} \ge 4 \quad (5b)$$

Therefore, α'_k and α^*_k can be regarded as two extended damping coefficients from which the robust stability of α_k can be estimated.

A damping ratio defined as [10]:

$$\xi_k = -\alpha_k / \sqrt{\alpha_k^2 + \beta_k^2} \quad (6)$$

should have positive value. Similarly to α'_k and α^*_k in (5), two extended damping ratios may be determined as (7) from the expectation $\bar{\xi}_k$ and standard deviation σ_{ξ_k}:

$$\xi'_k = \bar{\xi}_k - 4\sigma_{\xi_k} \quad (7a)$$

$$\xi^*_k = (\bar{\xi}_k - \xi_c)/\sigma_{\xi_k} \quad (7b)$$

To ensure the system dynamic performance, ξ'_k should not be less than a value ξ_C, i.e. $\xi'_k \ge \xi_C$, or $\xi^*_k \ge 4$. In this study, $\xi_C = 0.1$ [10].

3. Probabilistic sensitivity indices (PSIs)

Since the probabilistic nature of an eigenvalue is described by its expectation and variance, the probabilistic eigenvalue sensitivity will be comprised of the expectation sensitivity and the variance sensitivity. The sensitivity of an eigenvalue expectation can be computed similarly to that in conventional deterministic condition using:

$$\frac{\partial \lambda_k}{\partial \kappa_i} = \mathbf{W}_k^T \frac{\partial \mathbf{A}}{\partial \kappa_i} \mathbf{U}_k \quad (8)$$

where $\mathbf{A}$ is the system matrix. $\mathbf{W}_k$ and $\mathbf{U}_k$ are respectively the left and right eigenvectors of eigenvalue λ_k with $\mathbf{W}_k^T \mathbf{U}_k = 1$. κ_i stands for a parameter, such as a nodal voltage or a PSS parameter.

The sensitivity of an eigenvalue variance is derived from (3) and expressed in general as:

$$\frac{\partial C_{\gamma_k,\eta_k}}{\partial K_m} = \sum_{i=1, j=1}^{2N,2N} \left[C_{V_{i,j}} \left(\frac{\partial^2 \gamma_k}{\partial V_i \partial K_m} \frac{\partial \eta_k}{\partial V_j} + \frac{\partial \gamma_k}{\partial V_i} \frac{\partial^2 \eta_k}{\partial V_j \partial K_m} \right) \right] \quad (9)$$

where K_m stands for m-th PSS parameter. With γ_k and η_k in (9) denoting α_k or β_k for eigenvalue $\lambda_k = \alpha_k + j\beta_k$, C_{γ_k,η_k} in the left hand side of (9) may stand for variance C_{α_k,α_k}, C_{β_k,β_k} or C_{α_k,β_k}. Considering $\sigma^2_{\alpha_k} = C_{\alpha_k,\alpha_k}$, the sensitivity of standard deviation is simply:

$$\frac{\partial \sigma_{\alpha_k}}{\partial K_m} = \frac{1}{2\sigma_{\alpha_k}} \frac{\partial C_{\alpha_k,\alpha_k}}{\partial K_m} \quad (10)$$

Sensitivities of α'_k and α^*_k in (5) can be regarded as probabilistic sensitivity indices for damping represented by S'_{α_k} and $S^*_{\alpha_k}$ respectively as: