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A material derivative approach in design sensitivity analysis of three-dimensional contact problems

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Abstract

A shape design sensitivity analysis (DSA) and the optimization of a three-dimensional (3-D) contact problem is proposed using a material derivative approach. A penalty-regularized contact variational equation is differentiated with respect to the shape design parameter. A die shape DSA is also carried out by defining a design velocity field at rigid-body geometry. The material derivative that is consistent with the frictional return-mapping scheme is derived by using nonassociative plasticity. A linearized design sensitivity equation is solved without iteration by using a meshfree method at each converged load step. In order to improve the convergence behavior of the contact problem, a C^2-continuous contact surface is constructed from the scattered set of particles. The accuracy and efficiency of the proposed method is shown using two-dimensional and 3-D design examples of the DSA and optimization process.

Keywords: Design sensitivity analysis; Design optimization; Contact problem; Smooth contact surface; Meshfree method

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1. Introduction

Although many engineering applications include contact constraints, slow progress has been made in design sensitivity analysis (DSA) and optimization, especially as compared to the fast growth in structural DSA of noncontact problem. This is partly due to the complicated kinematics involved in contact analysis, and the theoretical depth required in variational inequality. The effect of contact constraint on structural performance must be taken into account in designing structural components that make contact with other parts. In the sheet metal stamping process, for example, a die shape design is critical to control workpiece shape after spring-back, to reduce wrinkling effects, and to remove the phenomena of necking. A die shape design parameter exerts its influence on structural performance through the contact constraint. In this

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paper, three-dimensional (3-D) contact DSA and optimization are developed by extending previous two-dimensional (2-D) research results (Kim et al., 2000).

The DSA of a contact problem has been approached from the mathematical perspective, in which the variational inequality is differentiated to obtain design sensitivity variational inequality. By understanding the variational inequality as a projection, Mignot (1976) proved that the projection onto the convex cone (constraint set) is directionally differentiable. Sokolowski and Zolesio (1991) derived a shape sensitivity formulation for the variational inequality from Mignot's result. They conclude that the solution to the variational inequality is directionally differentiable, and its shape sensitivity is a solution to another variational inequality, which is a projection onto a common convex set of tangential and orthogonal subspaces.

Because there is no mathematical proof for the existence of design sensitivity for nonlinear problems, the approximated variational equation or finite element matrix equation has been differentiated to accommodate many engineering applications. Spivey and Tortorelli (1994) presented a sensitivity formulation of the nonlinear frictionless contact problem for a beam, and optimized the geometry of the rigid surface. Antunez and Kleiber (1996) derived a sensitivity formulation of the contact problem using a flow approach to analyze the structure. Pollock and Noor (1996) developed a nonlinear dynamic sensitivity formulation using the discrete DSA method by differentiating the finite element matrix equation. Maniatty and Chen (1996) developed a design sensitivity formulation for the steady state metal-forming process using a semi-analytical adjoint variable method. Zhao et al. (1997) solved an unconstrained optimization problem to minimize the difference between the shape of the stamped workpiece and the desired shape. Their sensitivity equation requires an additional tangent stiffness matrix that is different from the one used in response analysis. Chung and Hwang (1998) proposed a method for transient forming process optimization. Since a semi-analytical method is used to compute the sensitivity coefficient, accuracy depends on the size of the design perturbation. Recently, Zabaras et al. (2000) applied die shape DSA to the 2-D metal-forming process. However, a general 3-D contact DSA method that includes a large deformation in the elastoplastic material and complicated frictional behavior is not reported in the literature. In this paper, shape DSA of a 3-D contact problem is developed using the material derivative approach. Instead of the variational inequality, a penalty-regularized variational equation is differentiated with respect to the structural shape and die shape design parameters. The material derivative that is consistent with the frictional return-mapping scheme is derived.

Discretization of the nonlinear equation and the design sensitivity equation is carried out using the meshfree method (Liu et al., 1995; Chen et al., 1996), where the structural domain is represented by a set of particles. The meshfree shape function is obtained from a set of supporting particles around an integration point to satisfy a reproducing condition, which exactly represents a certain order of polynomials. In contrast to finite element analysis, the construction of a shape function is independent of the mesh geometry in the meshfree method. Thus, this method is attractive for both a large deformation problem and a large shape-changing design problem, in which initially regular mesh can be significantly distorted during nonlinear analysis and during the shape optimization process. However, since the proposed approach is based on the continuum method, other discretization methods are easily applicable using the minimum implementation effort.

A piecewise-linear contact surface causes a significant amount of difficulty in the Newton-type iterative method because it lacks continuity across the surface boundary. From a computational point of view, a C^2-continuous surface is required to guarantee a continuous contact force across the boundary. A piecewise C^3-continuity is additionally required to provide a valid tangent stiffness matrix at each surface. In the finite element-based method, however, it is difficult to generate such regular surface patches. In this paper, a meshfree technique is used to produce a smooth surface from a set of scattered particles whose connectivity information is not provided in advance (Wang, 2000).

The frictional mechanism is modeled using a nonassociative plasticity (Michalowski and Mroz, 1978). Since the friction force and the contact location at the previous load step are required to calculate friction