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Design sensitivity analysis of sheet metal forming processes with a direct differentiation method

S.H. Kim[a], H. Huh[b,*]

[a]Vehicle Development and Analysis Team, Hyundai Motor Company, 772-1, Jangduk-Dong, Whasung-Si, Kyunggi-Do 445-706, South Korea
[b]Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, Science Town, Daejeon 305-701, South Korea

Abstract

A design sensitivity analysis scheme is proposed for an elasto-plastic finite element analysis with explicit time integration using a direct differentiation method (DDM). The direct differentiation scheme proposed deals with large deformation, an elasto-plastic constitutive relation considering the planar anisotropy, shell elements with reduced integration and complicated contact between the sheet and the dies. The design sensitivities with respect to the process parameter are calculated with direct analytical differentiation of the governing equation. The present DDM result is compared with the result obtained from a finite difference method (FDM) in sheet metal forming processes such as a cylindrical cup drawing process and a U draw-bending process for the verification of its reliability, accuracy and versatility. DDM results show good agreement with FDM results. Results fully demonstrate that sensitivity calculation with FDM is sometimes very dangerous since it could lead to a fatal pitfall and selection of the sensitivity scheme should be carefully done for specific problems. The analysis shows that the sensitivity of state variables provides useful information not only for the strain control but also for the inspection of defect initiation such as fracture or wrinkle. The result demonstrates that the sensitivity calculation scheme proposed is applicable to analysis and design of complicated sheet metal forming.

Keywords: Design sensitivity analysis; Direct differentiation method; Sheet metal forming process

1. Introduction

Design sensitivity analysis is widely used in mechanical problems in order to investigate the effect of the design variable on the objective system response quantitatively or to solve the design optimisation problem. Researches on the design sensitivity analysis have been started in the nonlinear solid mechanics field since the early part of 1980s. Recently, many studies are focused on the complicated nonlinear problem such as the collapse analysis and the forming analysis in order to obtain the optimum solution and to achieve the stable result.

The sheet metal forming process is influenced by many process parameters such as the die geometry, the blank shape, the blank holding force (BHF), the bead force, friction and so forth. In spite of its importance for determination of process parameters, the effect of process parameters is yet explored by the experience, the intuition or the time-consuming computer analysis such as incremental finite element methods for small modification during the process design. When the number of process parameters considered becomes larger, it becomes more difficult to determine the optimum values of parameters satisfying the design specifications without any problem during the forming process. The optimisation process requires more time to choose the optimum parameters by trial and error.

Design methods of process parameters have been developed since 1990s in the metal forming analysis with the finite element method and the design optimisation theory. While many achievements and applications have been done for optimisation in bulk forming processes, application of the theories to sheet metal forming processes remains very difficult because of its inherent complexity during the forming process.

In sheet metal forming processes, Kleiber et al. [1] carried out the sensitivity formulation with respect to the friction in a hemi-spherical punch forming process using the direct differentiation method (DDM). Naceur et al. [2] predicted the drawbead restraining force in square cup drawing and dashpot forming. They could not achieve accurate design for the optimum process parameters since the analysis has mainly incorporated with an inverse finite element method as an analysis tool. Ohata et al. [3] proposed a sweeping simplex method for calculating the height of the intermediate die of

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[*] Corresponding author.

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an oil pan. Lee and Huh [4] developed a parameter estimation algorithm that incorporated with an inverse finite element method and a DDM. The initial blank shape and optimum values of non-shape parameters were obtained. Hillmann and Kulbi [5] determined the optimum bead force and variable BHF with respect to the punch stroke with finite element analysis and the evolution strategy in complicated stamping processes. However, the application seems difficult to be performed since the simulation required tremendous time. Ghouati and Gelin [6] developed a determination system of process parameters with an explicit finite element analysis and a DDM. Yang et al. [7] calculated the design sensitivity and applied to the optimum design of the hydroforming process. Huh and Kim [8] has developed a parameter determination system using a rigid-plastic finite element analysis and an optimisation scheme. Doltsinis and Rodič [9] applied a sensitivity-based optimisation scheme to a superplastic forming process design.

In this paper, a design sensitivity analysis scheme is proposed with an elasto-plastic finite element method with explicit time integration and a DDM. The sensitivity information is utilised in the process optimisation of sheet metal forming. Direct differentiation is obtained considering large deformation, the elasto-plastic constitutive relation with the planar anisotropy, shell elements with reduced integration and complicated contact between the sheet and the dies. Design sensitivities with respect to the process parameter are obtained with the direct analytical differentiation of the equilibrium equation for the finite element analysis. The present result is compared with the result obtained with the finite differentiation method (FDM) in sheet metal forming processes such as a cylindrical cup drawing process and a U draw-bending process for the verification of its reliability, accuracy and versatility.

2. Finite element method

The direct analysis incorporates with an elasto-plastic finite element method with explicit time integration. The explicit analysis is now widely used in real industrial problems with the complicated contact. The analysis adopts the shell element proposed by Belytschko et al. [10] with reduced integration which is fortified by a hourglass control scheme following the Flanagan and Belytschko's work [11].

Integral form derived from the principle of virtual work can be approximated by spatial discretization with shell elements for a matrix equation at time $t = t^n$ as follows:

$$\mathbf{R}^n = \mathbf{M}^n\mathbf{a}^n - \mathbf{F}^n_{\text{ext}} + \mathbf{F}^n_{\text{int}} = \mathbf{0} \tag{1}$$

where $\mathbf{a}^n$, $\mathbf{M}^n$, $\mathbf{F}^n_{\text{ext}}$ and $\mathbf{F}^n_{\text{int}}$ are the acceleration vector, the lumped mass matrix, the external force vector neglecting contribution of the body force and the internal force vector at the nodal points, respectively. The expressions of each variable can be found in many related literatures.

Explicit integration of the governing equation in the time domain is performed with the central difference method. The velocity and the coordinate vectors at the current time step are calculated as follows:

$$\mathbf{v}^{n+(1/2)} = \mathbf{v}^{n-(1/2)} + \mathbf{a}^n\Delta t^n \tag{2}$$

$$\mathbf{x}^{n+1} = \mathbf{x}^n + \mathbf{v}^{n+(1/2)}\Delta t^{n+(1/2)} \tag{3}$$

where $\Delta t^{n+(1/2)}$ is expressed as

$$\Delta t^{n+(1/2)} = \frac{\Delta t^n + \Delta t^{n+1}}{2} \tag{4}$$

The sheet metal is assumed to be elasto-plastic, possess the planar anisotropy and obeys Hill's quadratic yield criterion [12] and its associated flow rule. In order to calculate the stress tensor in the given time increment Δt^n, a radial return mapping scheme proposed by Simo and Taylor [13] is adopted.

In order to treat contact phenomena between the tool and the blank, a penalty method is adopted. The die shapes are discretized to the four-node shell element and the tools are assumed to be rigid. If a slave node penetrates through the master segment as much as g, the normal contact force is imposed as

$$\mathbf{f}_{\text{s}} = -gk\mathbf{n} \tag{5}$$

where g is the gap function which is the penetration amount, k the interfacial stiffness and $\mathbf{n}$ the normal vector defined at the contacting point $(\xi_{\text{c}}, \eta_{\text{c}})$ in the master segment.

3. Design sensitivity analysis

A scheme for the design sensitivity analysis has been developed using a DDM for the process design in sheet metal forming. The scheme deals with large deformation with shell elements, the elasto-plastic constitutive relation for a planar anisotropic material, and the complicated contact condition.

The first-order differentiation of the governing equation (1) for the finite element analysis with respect to the design variables $\mathbf{p}$ gives

$$\frac{\mathrm{d}\mathbf{R}^n}{\mathrm{d}\mathbf{p}} = \frac{\mathrm{d}(\mathbf{M}^n\mathbf{a}^n)}{\mathrm{d}\mathbf{p}} - \frac{\mathrm{d}\mathbf{F}^n_{\text{ext}}}{\mathrm{d}\mathbf{p}} + \frac{\mathrm{d}\mathbf{F}^n_{\text{int}}}{\mathrm{d}\mathbf{p}} \cong 0 \tag{6}$$

After solving the governing equation for the sensitivity analysis, the design sensitivity with respect to the design variable is obtained.

$$\frac{\mathrm{d}\mathbf{a}^n}{\mathrm{d}\mathbf{p}} = \mathbf{M}^{-1}\left(\frac{\mathrm{d}\mathbf{F}^n_{\text{ext}}}{\mathrm{d}\mathbf{p}} - \frac{\mathrm{d}\mathbf{F}^n_{\text{int}}}{\mathrm{d}\mathbf{p}} - \frac{\mathrm{d}\mathbf{M}^n}{\mathrm{d}\mathbf{p}}\mathbf{a}^n\right) \tag{7}$$

The sensitivities of the velocity vectors and the coordinate vectors are updated by differentiating the central difference scheme (2) and (3) for the time integration as follows:

$$\frac{\mathrm{d}\mathbf{v}^{n+(1/2)}}{\mathrm{d}\mathbf{p}} = \frac{\mathrm{d}\mathbf{v}^{n-(1/2)}}{\mathrm{d}\mathbf{p}} + \frac{\mathrm{d}\mathbf{a}^n}{\mathrm{d}\mathbf{p}}\Delta t^n \tag{8}$$