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Sensitivity analysis and modification of structural dynamic characteristics using second order approximation

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Abstract

This paper presents a formulation in the form of an inverse eigen value problem for modification of vibration behavior of structures. The proposed method which is based on the second order approximation in Taylor expansion is expressed in terms of variables relating to stiffness or mass matrix parameters in a finite element formulation. An initial sensitivity analysis identifies the regions within the structure where the modifications would yield the required changes in the structures dynamic characteristics. An algorithm is developed which allows efficient modification of structural dynamics characteristics without iterations. These modifications are conducted locally so that only elemental stiffness and matrices are affected. The algorithm is applied to four case studies and it is found that large modification of natural frequencies of up to 10% can be realized with an induced error of less than 5% for truss structures, and less than 3% for plane problems.

1. Introduction

The common industrial practice for optimising the vibration behavior of structures is to conduct a series of modifications on the FEA simulations of the structure in order to achieve the required eigenfrequencies. This approach, known as the *forward variation* approach or *design load analysis cycle* is extremely time consuming, expensive and rarely yields to an *optimum* solution. The vibration optimisation problem can be defined as an inverse engineering problem. The inverse engineering refers to problems where the desired response of the system is known or decided but the physical system is unknown. These problems are difficult because a unique solution is rarely possible.

The current state-of-the-art in the inverse approach to the vibration problem is only limited to structures modelled using simple linear springs, dampers and point masses. Very little attention has been given to formulating the inverse eigenvalue problem for two and three dimensional and higher order finite elements which are most commonly used in simulation of real structures. Optimisation of vibration characteristics is defined as an inverse eigenvalue problem or problem of designing systems in order to produce the desired response. To eliminate the need to re-analyse the whole structure modelled by finite elements, an inverse approach is required in order to find the exact modified parameters in various finite element formulations which yield the required natural frequencies. Early work in tackling the inverse eigenvalue problem by other researchers [1,2] utilised the 1st order terms of Taylor's series expansion and is based on Rayleigh's work. Others such as Chen and Garba [3] used the iterative method to modify structural systems. Recently Baldwin and Hutton [4] presented a detailed review of structural modification techniques. These were classified into categories of the techniques based on small modification, techniques based on localised modification and those based on modal approximation.

Further research on structural modification was carried out by Tsuei et al. [5–7] who presented a method of shifting the desired eigenfrequencies using the forced response of the system. The method is based on modification of either the mass or stiffness matrix by treating

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the modification of the system matrices as an external forced response. This external forced response is formulated in terms of the modification parameters, thus creating a modified eigenvalue problem. More recently Zhang and Kim [8] investigated the use of mass matrix modification to achieve desired natural frequencies. McMillan and Keane [9] investigated a method of shifting eigenfrequencies of a rectangular plate by adding concentrated mass elements.

Sivan and Ram [10,11] extended further the research on structural modification by studying the construction of a mass spring system with prescribed natural frequencies. They obtained stiffness and mass matrices using the orthogonality principles. They [12] developed a new algorithm based on Joseph's work [13] which involves the solution of the inverse eigen value problem. In the last few years the work on the inverse problem by Gladwell [14] started to be taken seriously by engineers and researchers interested in this field of engineering. Mottershead [15] also considered the problem of resonance in the forced vibration of machines and structures by the design of physical modifications to achieve targeted natural frequencies. His technique of achieving the required system include structural modifications by adding a point mass, a grounded spring or by a spring connecting two co-ordinates. Li et al. [16] considered optimising dynamic behavior of a multi body system by conducting modifications on its mass and stiffness matrices.

The above techniques have predominantly been applicable to discrete systems made up of simple linear spring and mass elements. Even with these simple elements the problem of mapping of the 'physically viable' stiffness and mass matrix to a real structure has not been fully resolved and the challenging problem of applying the inverse vibration problem to continuous finite elements has not yet been addressed. The method proposed in this paper is based on a matrix treatment procedure for modifying stiffness and mass matrices of the finite elements most commonly used in modelling structures as continuous systems. Our earlier work [17,18] was focused on bar and beam elements. This was then extended [20,21] to two dimensional elements. The new formulation significantly improves the previous work by using the second order Taylor approximation in the inverse formulation. Moreover, the proposed technique conducts the modification on the mass and stiffness matrices at a local level thence reducing the computational effort considerably.

2. Theory

2.1. First derivative of eigenvalue

Consider the rate of change of eigenvalue with respect to design variables [19]:

$$\varsigma' = \{y\}^T[K]'\{y\} - \varsigma\{y\}^T[M]'\{y\} \quad (1)$$

where ζ is the eigenvalue, $\{y\}$ is the eigenvector and $[k]$, $[m]$ are stiffness and mass matrices of the system respectively.

If we choose b_j as the finite elements physical or geometrical property, where j is the element number, then the derivatives with respect to b_j of the terms in the elements of matrices not containing b_j become zero. Using this reasoning, we showed in our previous work [20] that the vectors and matrices in the above equation can be changed to a reduced form and so the rate of change of eigenvalue with respect to b_j becomes:

$$\frac{\partial \overset{(i)}{\varsigma}}{\partial b_j} = \frac{\partial K_{rs}}{\partial b_j}\overset{(i)}{y_r}\overset{(i)}{y_s} - \overset{(i)}{\varsigma}\frac{\partial M_{rs}}{\partial b_j}\overset{(i)}{y_r}\overset{(i)}{y_s} \quad (2)$$

The index i denotes the i^{th} eigenvalue and eigenvector and the suffices r,s are summed over the numbers of the rows and columns of the stiffness and mass matrices of the particular element under consideration. They can be found by using the connectivity matrix [B]. The global numbers of nodes of the element j will be located in the row j of the matrix [B].As an example, for the case of plane truss with linear elements the r and s indices are determined from:

$$r,s = (2B_{(j1)}-1),(2B_{(j2)}),(2B_{(j2)}-1),(2B_{(j2)}). \quad (3)$$

2.2. Second derivative of eigenvalue

The second derivative of eigenvalue can be derived by differentiating Eq. (2) with respect to b_k, where b_k is another geometrical or physical property of the system (as a special case, it can be the same as b_j):

$$\frac{\partial^2 \overset{(i)}{\varsigma}}{\partial b_j \partial b_k} = 2\frac{\partial \overset{(i)}{y_r}}{\partial b_k}\left(\frac{\partial K_{rs}}{\partial b_j} - \overset{(i)}{\varsigma}\frac{\partial M_{rs}}{\partial b_j}\right)\overset{(i)}{y_s} + \overset{(i)}{y_r}\left(\frac{\partial^2 K_{rs}}{\partial b_j \partial b_k} - \overset{(i)}{\varsigma}\frac{\partial^2 M_{rs}}{\partial b_j \partial b_k} - \frac{\partial \overset{(i)}{\varsigma}}{\partial b_k}\frac{\partial M_{rs}}{\partial b_j}\right)\overset{(i)}{y_s}. \quad (4)$$

All the terms in the above equation are known except $\frac{\partial y^{(i)}}{\partial b_k}$, the derivative of eigenvector.

2.3. First derivative of eigenvector

We start with the equation of motion of the system subjected to undamped free vibrations:

$$(K_{mn} - \overset{(i)}{\varsigma}M_{mn})\overset{(i)}{y_n} = 0 \text{ (m,n = 1,2,...N)} \quad (5)$$

where N is the total degree of freedom of the system. Differentiating with respect to b_k gives:

$$(K_{mn} - \overset{(i)}{\varsigma}M_{mn})\frac{\partial \overset{(i)}{y}}{\partial b_k} = -\frac{\partial(K_{mn} - \overset{(i)}{\varsigma}M_{mn})}{\partial b_k}\overset{(i)}{y}. \quad (6)$$