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Insurance: Mathematics and Economics 35 (2004) 679–690
INSURANCE MATHEMATICS & ECONOMICS
www.elsevier.com/locate/econbase

A Malliavin calculus approach to sensitivity analysis in insurance

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Received 23 July 2003; received in revised form 9 August 2004; accepted 13 August 2004

Abstract

Using the Malliavin calculus on Poisson space and a method initiated by Fournié et al. [Fournié, E., Lasry, J.M., Lebuchoux, J., Lions, P.L., Touzi, N., 1999. Applications of Malliavin calculus to Monte Carlo methods in finance. Finance Stochastics 3, 391–412.] for continuous financial markets, we compute the probability density of risk reserve processes and the sensitivities of probabilities of ruin at a given date for insurance portfolios under interest force. The simulation graphs provided show that this method is computationally more efficient than the standard approximation of derivatives by finite differences.

JEL classification: G22

IBC Classification: IM10; IM13; IM40; IM20

AMS Classification: 91B30; 91B70; 60H07

Keywords: Probabilities of ruin; Reserve processes; Sensitivity analysis; Malliavin calculus

1. Introduction

In Norberg (2002) a method based on differential equations is proposed for the computation of sensitivities of conditional expected values of reserve processes in life insurance. In this paper we present a sensitivity analysis with respect to a parameter ζ for expectations of the form $E[h(U_\zeta(T))]$, where $U_\zeta(T)$ is the value at time T of a risk reserve process, ζ represents the initial reserve x or the interest rate r, and h is a not necessarily smooth, arbitrary integrable function. In particular when h is the indicator function $1_{(-\infty,\zeta]}$, this corresponds to the density

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0167-6687/$ – see front matter
doi:10.1016/j.insmatheco.2004.08.003

of ruin probabilities at a given date. Our method relies on the Malliavin calculus, which has been recently applied to numerical computations of price sensitivities of financial derivatives in continuous markets (Fournié et al., 1999) and in a market with jumps (El Khatib and Privault, 2004).

In models with interest force, probabilities of ruin at a given date have densities with respect to the Lebesgue measure, and we present a formula that allows for faster and more accurate numerical computation of this density. This method is also applied to compute the sensitivity of probabilities of ruin at a given date with respect to the initial reserve x and the interest rate parameter r. More precisely we will compute derivatives of the form

$$\frac{\partial}{\partial \zeta} E[h(U_\zeta(T))],$$

where

$$U_\zeta(T) = g(\zeta) + \int_0^T f_\zeta(t)\,\mathrm{d}X(t),$$

$(X(t))_{t\in[0,T]}$ is a compound Poisson process representing the number of claims occurring in $(0, T]$ and ζ is a parameter (initial reserve x, or interest force r). Such derivatives may be expressed as

$$\frac{\partial}{\partial \zeta} E[h(U_\zeta(T))] = E\left[\left(\partial_\zeta g(\zeta) + \int_0^T \partial_\zeta f_\zeta(t)\,\mathrm{d}X(t)\right) h'\left(U_\zeta(T)\right)\right].$$

However, this expression makes sense only when h is differentiable, in particular h cannot be an indicator function, hence the above expression cannot be used for ruin probabilities. Alternatively this derivative can be estimated by finite differences:

$$\frac{1}{2\varepsilon} E[h(U_{\zeta+\varepsilon}(T)) - h(U_{\zeta-\varepsilon}(T))], \tag{1}$$

but this approximation yields poor convergence results when combined with Monte Carlo methods, as shown in the simulations of Section 5. Instead of (1) we will show that $(\partial/\partial\zeta)E[h(U_\zeta(T))]$ can be expressed as

$$\frac{\partial}{\partial \zeta} E[h(U_\zeta(T))] = E[W_\zeta h(U_\zeta(T))], \tag{2}$$

where W_ζ is a random variable called a weight which is explicitly computable and independent of h. Expression (2) above yields a substantial improvement over the finite difference method (1) in the precision and speed of Monte Carlo numerical simulations. This formula is obtained by integration by parts on the Poisson space, using a gradient operator which acts on the Poisson jump times of $(X(t))_{t\in\mathbb{R}_+}$. Our approach actually requires the considered random variable $U_\zeta(T)$ to be sufficiently smooth to be in the domain of D_w with $D_w U_\zeta(T) \neq 0$, a.s. These assumptions are linked to the existence of density with respect to the Lebesgue measure for the probability law of $U_\zeta(T)$. For example, D_w vanishes on functions of the Poisson random variable $N(T)$, which do not have a density, and this excludes in particular models without interest force from our analysis.

We proceed as follows. Section 2 contains preliminaries on the Malliavin calculus on Poisson space and on the differentiability of random functionals. In Section 3 we present the integration by parts formula which is the main tool to compute sensitivities (i.e. derivatives with respect to ζ) using a random variable called a weight. The model and explicit computations for reserve processes are presented in Section 4. In Section 5 we provide numerical simulations which demonstrate the efficiency of the Malliavin approach over finite difference methods.