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Continuum shape sensitivity analysis of a mixed-mode fracture in functionally graded materials

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Abstract

This paper presents two new methods for conducting a continuum shape sensitivity analysis of a crack in an isotropic, linear-elastic functionally graded material. These methods involve the material derivative concept from continuum mechanics, domain integral representation of interaction integrals, known as the *M*-integral, and direct differentiation. Unlike virtual crack extension techniques, no mesh perturbation is needed to calculate the sensitivity of stress–intensity factors. Since the governing variational equation is differentiated prior to the process of discretization, the resulting sensitivity equations are independent of approximate numerical techniques, such as the meshless method, finite element method, boundary element method, or others. Three numerical examples are presented to calculate the first-order derivative of the stress–intensity factors. The results show that first-order sensitivities of stress intensity factors obtained using the proposed method are in excellent agreement with the reference solutions obtained using the finite-difference method for the structural and crack geometries considered in this study.

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Keywords: Crack; Functionally graded materials; Interaction integral; *M*-integral; Linear-elastic fracture mechanics; Shape sensitivity analysis; Material derivative

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1. Introduction

Functionally graded materials (FGMs) that possess spatially varying microstructure and mechanical/thermal properties are essentially multi-phase particulate composites, engineered to meet a predetermined functional performance [1,2]. In recent years, various theoretical, computational, and experimental studies have been conducted to understand the fracture behavior of FGMs. A collection of technical papers, published in [3] reflects state-of-the-art research into FGM fracture. A major component of such studies involves calculating the crack-driving forces in FGMs both accurately and efficiently. Consequently, various numerical methods have been developed for calculating stress–intensity factors (SIFs), and can be found in the above-mentioned literature. More recently in 2003, Rao and Rahman [4,5] developed two interaction integrals for the mixed-mode fracture analysis of cracks in isotropic and orthotropic FGMs. In contrast to existing methods, it is not necessary to perform integration along the crack face of the discontinuity. Hence, the interaction integral method is simpler and more efficient than previously existing methods.

While past studies demonstrate a better understanding of FGM fracture, they also indicate areas of future development. For example, in many fracture mechanics applications, either the derivatives or sensitivities of SIF with respect to crack size are needed for predicting stability and arresting crack propagation in FGM. Another major use of SIF derivatives is in the reliability analysis of cracked structures. For example, the first- and second-order reliability methods [6], frequently used in probabilistic fracture mechanics [7–13], require the gradient and Hessian of the performance function with respect to crack length. In a linear-elastic fracture, the performance function builds on SIF. Hence, both first- and second-order derivatives of SIF are needed for probabilistic analysis of FGMs.

For predicting sensitivities of SIF under a mode-I condition, some methods have already appeared for homogenous materials. In 1988, Lin and Abel [14] employed a virtual crack extension technique [15–18] and the variational formulation in conjunction with the finite element method (FEM) to calculate the first-order derivative of SIF for a structure containing a single crack. Subsequently, Hwang et al. [19] generalized this method to calculate both first- and second-order derivatives for structures involving multiple crack systems, an axisymmetric stress state, and crack-face and thermal loading. However, these methods require mesh perturbation, a fundamental requirement of all virtual crack extension techniques. For second-order derivatives, the number of elements surrounding the crack tip affected by mesh perturbation has a significant effect on solution accuracy. To overcome this problem, Chen et al. [20–22] recently applied concepts from shape sensitivity analysis to calculate the first-order derivative of SIFs. In this new method, the domain integral representation of the *J*-integral (mode-I) or the interaction integral (mixed-mode) is invoked and the material derivative concept from continuum mechanics is then used to obtain the first-order sensitivity of SIFs. Since the governing variational equation is differentiated before discretization, the resulting sensitivity equations are independent of any approximate numerical techniques, such as FEM, the boundary element method, or others. However, most of the analytical methods discussed above are developed for the sensitivity analysis of cracks in homogenous materials. Only recently, Rao and Rahman [23] developed a sensitivity analysis method for a crack in an isotropic, linear-elastic FGM under mode-I loading conditions. Hence, there is a clear need to develop similar sensitivity equations for mixed-mode loading conditions.

This paper presents a new method for predicting the first-order sensitivity of mode-I and mode-II stress–intensity factors, K_{I} and K_{II}, respectively, for a crack in an isotropic, linear-elastic FGM. This method uses the material derivative concept from continuum mechanics, domain integral representation of an interaction integral, known as the *M*-integral, and direct differentiation. Unlike virtual crack extension techniques, no mesh perturbation is needed in the proposed method to calculate the sensitivity of stress–intensity factors. Since the governing variational equation is differentiated prior to the process of discretization, resulting sensitivity equations are independent of such approximate numerical techniques as the meshless method, the finite element method, the boundary element method, or others. Numerical examples are presented to calculate the first-order derivative of the *M*-integral and stress–intensity factors using the pro-

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