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Thermomechanically coupled sensitivity analysis and design optimization of functionally graded materials

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Abstract

This paper presents a systematic numerical technique for performing sensitivity analysis of coupled thermomechanical problem of functionally graded materials (FGMs). General formulations are presented based on finite element model by using the direct method and the adjoint method. In the modeling of spatial variances of material properties, the graded finite element method is employed to conduct the heat transfer analysis and structural analysis and their sensitivity analysis. The design variables are the volume fractions of FGMs constituents and structural shape parameters. The design optimization model is then constructed and solved by the sequential linear programming (SLP). Numerical examples are presented to demonstrate the accuracy and the applicability of the present method.

1. Introduction

In recent years, functionally graded materials (FGMs) have been shown to possess superior advantages when employed in high temperature environment. Tanigawa [35], Koizumi [15], Suresh and Mortensen [31], Noda [27] reviewed the development history of FGMs from the aspects of heat conduction analysis, thermal stress related problems, research activities and manufacturing techniques. One of the most important characteristics of FGMs is that they are designable. One can design the structure of FGMs both to resist the severe thermal loading due to high temperature and to maintain the structural rigidity and stiffness in a desired way by tailoring design parameters.

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In respect of sensitivity analysis and design optimization, the research for general themoelastic problems has placed a solid foundation for the investigation of FGMs where both the heat conduction and the structural analysis can be taken into account at the same time. The work of sensitivity analysis started from the middle of 1980s. Meric [23,24] deduced the sensitivity equations for static problem in which the design variables were the material properties and the parameters of external loads. Dems and Mroz [8], Meric [25] investigated the sensitivity analysis for static and quasi-static problems with respect to size and shape design variables. Tortorelli et al. [36] presented the sensitivity analysis with adjoint method for the coupled constitutive model. Hou et al. [12], discussed the shape sensitivity analysis with both the direct and the adjoint method for static problem in conjunction with the finite element method. The similar work for 3D problem was conducted by Yang [39]. Most of the contributions mentioned above are based on the continuum model where the sensitivity equations were deduced directly from the partial differential equations. Another approach is to derive the sensitivity equations based on discrete model, in which the differential equations of a problem are discretized firstly in space by numerical techniques, such as the finite element method and then the sensitivity analysis is performed based on the discretized equations. The formulations on the discrete model are concise and facilitate the program implementation. Based on the finite element model, Chen et al. [3] and Liu [20] presented the direct method and the adjoint method, respectively, for both static and quasi-static problems. On the other hand, different kinds of techniques have been applied to design optimization for general thermoelastic problem. The design optimization work based on sensitivity analysis were given by Meric [25], Hou et al. [12], Chen et al. [3] and Liu [20] for general thermoelastic problems, Michaleris et al. [26] for weldment problem and Autio [1] for laminated plates. By using boundary integral equation, Lee and Kwak [17,18] investigated the shape design optimization for two-dimensional and axisymmetric problems. Kok et al. [16] employed response surface method in their research. Li et al. [19] performed the topology design optimization for thermoelastic problem by using evolutionary structural optimization (ESO). Xu and Grandhi [38] studied the multi-points approximation techniques in the design optimization of thermostructural problems.

Due to the benefits of the application of FGMs, attentions have been focused on the design optimization of FGMs (see Tanigawa [35], Markworth et al. [21], Markworth and Saunders [22] and Noda [27]). Tanaka et al. [32–34] fulfilled a series of work on design optimization of volume fractions of FGMs by using the incremental finite element analysis and the direct sensitivity method. Cho and Ha [6,7] performed the design optimization of 1D and 2D volume fraction of FGMs. They conducted the sensitivity analysis by using the global finite difference method (FDM) and then employed the interior penalty function method to solve the optimal model. Huang et al. [13] studied design optimization through employing the strength and the energy storage capability of the FGMs structure as two objectives. Trueltaub [37] discussed the optimal control and optimization of FGMs in which the objective function can reflect the control problem or the optimization problem or both by tailoring its parameters. In his study, the sensitivity analysis was performed based on the continuum model and optimal criterion method was employed. In addition to the above work, some researchers have presented different schemes on the subject. Markworth and Saunders [22] proposed a simple optimization model for one-dimensional problem. Based on the analytical solutions, Ootao et al. investigated the application of some bionic algorithms to the design optimization of FGMs, such as the neural network method for the hollow sphere [28] and the hollow circular cylinder [29], and the genetic algorithm for the laminated composite plate [30].

In this paper, we apply the techniques of structural optimization for thermoelasticity [3,20] to the design optimization for FGMs which treat the volume fraction and structural shape design variables. Firstly, based on finite element model, sensitivity analysis schemes, in conjunction with the direct method and the adjoint method, are presented for both static and quasi-static problems. And then the graded finite element method proposed by Kim and Paulino [14] is employed to conduct the heat transfer and structural analysis. The advantage of the graded finite element method is its compatibility with the prob-