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A simplified force-based method for the linearization and sensitivity analysis of complex manipulation systems

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Abstract

In this paper a method is proposed to efficiently linearize the geometry of complex multibody systems by exploiting the kinetostatic dualism, i.e. formulating the linearization in terms of the force transmission. In this setting, an algorithm is introduced by which one can perform the linearization of the transmission behavior from any number of geometric parameters to the motion of a six-degree-of-freedom end-effector by applying six unit loads to the end-effector and determining internal forces. Moreover, applications in first-order error analysis, calculation of the stiffness matrix, and calibration of manipulators are proposed. Examples of both serial and fully parallel manipulators are presented.

Keywords: Kinematics; Multibody system; Kinetostatic method; First-order error analysis; Stiffness; Calibration; Parallel manipulator

1. Introduction

The analysis of manufacturing and assembly errors of manipulators is a topic that is highly relevant for practical applications because the magnitude of these errors is directly coupled to the total cost of production of the manipulator. In this setting, there exist intensive studies on how to estimate the error of certain moving points, e.g. the tool center point, in terms of the errors in the components of the mechanism [1–5], as well as how to allocate cost-optimal tolerances to a mechanism [6,7]. In this paper, a new approach to estimate the first-order influence of geometric errors on target quantities is suggested in which linearization is performed by considering the force transmission of the manipulator. This enables one to obtain a comprehensive model of linearized geometric sensitivities at a low computational cost.

1.1. Review of the literature

Error analysis for serial manipulators is relatively easy because one can establish an analytical expression for the direct kinematics which maps the generalized joint and link coordinates to the spatial displacements of

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the end-effector. There are numerous methods to parameterize the direct kinematics, where the approach of Denavit and Hartenberg [8] is the most popular one. Once one has a closed-form expression for the direct kinematics, one can take derivatives of it with respect to the geometric parameters one is interested in and use these as sensitivity coefficients. In general, one introduces the sensitivity parameters in such a way that they vanish at the nominal configuration. This is always possible by introducing corresponding constant offsets where necessary. For example, consider a robot involving an universal joint, and assume that the sensitivity to errors in the fulfillment of the intersection property of the axes is to be analyzed. This can be done by adding a parameter for the normal distance between the joint axes which is zero in the nominal design, and with respect to which the partial derivative will yield the sought sensitivity. However, such a method for sensitivity analysis results in a model with a significant overhead. Examples of such models for joints are presented e.g. by Brisan et al. [1] and Song et al. [4].

Sensitivity computations have also been investigated in the framework of clearance analysis, where two focuses exist. One focus is to determine the geometric variations of an entity of interest (e.g. tool center point, end-effector) for given variations of the joint reference parameters (center, alignment axis) within the joint clearance envelopes. Here, a first-order sensitivity analysis as described in this paper is sufficient. Previous authors that have addressed this topic are [9–12], were a method of force transmission similar to the one addressed in this paper is presented. However, the method in this paper is more comprehensive in the sense that it treats the problem of sensitivity in its more general form, including parameter sensitivity and stiffness analysis. Moreover, the paper is based on a general method of using force transmission to evaluate general Jacobians, which was already published in [13] and which provides a generic formulation for all kind of force-based sensitivity analyses including those in [9–12]. A second focus of clearance analysis is to regard the effect of clearance on dynamic quantities such as forces or on higher-order kinematics such as accelerations or limit positions. For such analyses, some approaches have been suggested which allows one to use standard kinematic code, such as the *equivalent clearance link* [28] model or the probabilistic method introducing an *effective link-length* [29]. However, such analyses require second-order or even higher-order derivatives which are outside of the scope of this paper and are moreover practically restricted to one-degree-of-freedom mechanisms. Such higher-order sensitivities might be a topic of future research.

A linearization method for complex mechanisms using the kinetostatic dualism and the concept of *kinematical differentials* to efficiently set up the equations of motion of multibody systems has been proposed by Kecskeméthy and Hiller [13]. Using this method, all required partial derivatives can be described solely by using the kinematic transmission functions for position and velocity, as well as the force transmission function of the system. Based on these transmission functions, an algorithm is formulated for generating the Jacobian and the equations of motion through multiple evaluations of the kinematic transmission functions for certain pseudo input velocities and accelerations. The corresponding algorithms are denoted as *kinematical differentials* for the case of the pure kinematic transmission function [14] and *kinetostatic approach* for the case of use of force transmission [13]. Later, Lenord et al. [15] showed that kinematical differentials may be applied also to more general interdisciplinary systems which also involve hydraulic components by using an exact linearization through the kinematical differentials for the determination of the velocity linearization and numerical differentiation for the calculation of the stiffness matrix of the hybrid system. Other authors studied the determination of the stiffness matrix for complex multibody systems using explicit symbolic derivatives [16,17], taking into account the stiffness of the actuators and the stiffness of special components. These approaches however require numerous computational steps when many sensitivity parameters are involved, thus, making alternative approaches necessary, as presented in this paper.

The rest of the paper is organized as follows: In Section 2 the theoretical background of the proposed approach for linearizing geometric dependencies in manipulator systems is presented. Here, first the kinetostatic transmission method is reviewed, and the notion of virtual joints is introduced by which the force-based method can be used to linearize the manipulator. Thereafter, some applications of the linearization of multibody systems, e.g. error analysis and determination of the stiffness matrix, are considered. In Section 3, simulation results for examples are presented, and in the last section the paper is closed with a number of conclusions.