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Design sensitivity analysis of nonlinear structures subjected to thermal loads

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Abstract

This work presents a simple and efficient methodology for sensitivity analysis of geometrically nonlinear structures subjected to thermo-mechanical loading in regular and critical states. Using the effective strain approach, the path-following methods and the algorithms for critical point computation developed originally for finite element analysis of mechanically loaded structures are modified to include the thermal effects. The general expressions for sensitivity computation of displacements, stresses and nonlinear critical loads are obtained through the differentiation of the finite element equations. The practical implementation of the sensitivity analysis in a finite element code employing the Analytical, Semi-Analytical and Refined Semi-Analytical approaches is discussed in detail. Finally, a set of numerical examples is used to validate the proposed methodology.

Keywords: Design sensitivity analysis; Structural optimization; Thermal loads; Nonlinear structures

1. Introduction

Several structures and mechanical components are subjected simultaneously to mechanical and thermal loading. It is well known that a temperature increase causes displacements and strains in structures whose expansion is not constrained, and that the so-called thermal stresses are generated when the free thermal expansion is restrained by supports or friction. Practical experience has shown that a temperature increase can create enough compressive stresses to cause buckling of slender structures. Therefore, thermal buckling is an important issue in the design of heated structures, like columns, pipelines, plates and shells.

The design optimization of structures including thermal buckling constraints has attracted much attention in the recent years, see [1] and references therein. However, most of these works are based on small displacement analysis and linearized buckling procedures, which limits the applicability of the proposed optimization formulations. As a matter of fact, the optimization of slender structures can lead to severe instability problems, including imperfection sensitivity and modal interaction [2]. It was observed that stability problems tend to occur when the optimization is based on the assumption of linear structural behavior [3].

In order to avoid these problems, the geometrically nonlinear effects should be considered in the structural analysis and the optimum design formulation should include an appropriate set of stability constraints [4]. It is also important to include the effects of load and geometry imperfections in the optimization model [5,6]. Generally, the presence of initial imperfections eliminates the bifurcation points, yielding either limit points for asymmetrical and unstable symmetric bifurcations or stable paths without critical points for stable-symmetric bifurcations [7]. In the latter case, large deflections can arise and appropriate displacement constraints should be included in the optimization model.

It should be noted that a uniform temperature field in a structure without geometric imperfections will generally

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preserve the critical states typical of perfect structures. On the other hand, a nonuniform field with a temperature gradient in the transverse direction of beams, plates and shells generates initial deflections which can eliminate the critical states even for perfect structures. Therefore, the effects of thermally induced bending should also be considered either alone or combined with geometric imperfections.

In this work, the thermal effects are included in the finite element formulation through the effective strain concept, leading to a simple and generic procedure for computation of the internal force vector and stiffness matrix of different finite elements [8]. The analysis procedure presented here can handle structures under pure mechanical loading, pure thermal loading and combined thermo-mechanical loading.

The determination of the load-carrying capacity of structures subjected to mechanical loads and temperature variations require the use of robust methods to trace the nonlinear equilibrium paths and to perform the critical point computation. These methods will be discussed here with focus on the relevant aspects to the sensitivity computation. Both perfect and imperfect structures will be considered.

The most efficient algorithms for structural optimization, as the Sequential Quadratic Programming (SQP) method, require the gradients of the objective and constraint functions to compute the search direction at each step of the optimization process [9,10]. Errors in computation of these gradients can degrade the performance of the optimization algorithm and lead to severe convergence problems. The gradients of the constraint functions depend on the derivatives (sensitivities) of structural responses, as displacements, stresses, and critical loads, with respect to the design variables. Therefore, the use of efficient and accurate procedures to perform sensitivity analysis is paramount to the success of the structural optimization process. In addition, sensitivity analysis has also other important applications, as parameter identification [11] and structural reliability analysis [12].

It is important to note that this paper is concerned only with the sensitivity of the structural responses, but methods to compute the sensitivity of the temperature field are readily available [13,14]. Therefore, in this work it is assumed that the temperature field and its sensitivity were previously computed and given to the structural analysis program as input data.

The general expressions for computation of design sensitivities in regular and critical states are obtained through the direct differentiation of the finite element equations. This approach leads to expressions required to the computation of displacements, stresses and nonlinear critical (limit and bifurcation) loads for both size and shape variables. The adjoint approach will also be applied to the computation of sensitivity of the nonlinear critical loads [15,16]. It results in a more efficient procedure than the direct one, but it will be shown that it can be used only for symmetric bifurcation points.

According to the dependence of the temperature field on the design variables, the sensitivities can be classified as coupled or uncoupled [1]. Most of the previous works are focused on the uncoupled case [15,6]. On the other hand, this work focuses on the coupled case and the dependence of the temperature field on the design variables is accounted for. It will be shown that the modifications in the sensitivity expressions required to consider the temperature variations are very similar to those performed in the path-following methods and in the procedures for critical point computation. The expressions for uncoupled sensitivities are easily obtained from the general expressions eliminating the terms related to the sensitivity of the temperature field.

The sensitivity formulation presented in this work can be applied to structures subjected to pure mechanical loading, pure thermal loading and combined thermo-mechanical loading. The procedure is independent of the element formulation and handles both perfect and imperfect structures in regular and critical states.

The expressions for sensitivity computation depend on the derivatives of the element vectors and matrices with respect to the design variables. These derivatives are computed here using the Analytical, Semi-Analytical and Refined Semi-Analytical Methods [17–20]. Numerical examples are presented to validate the proposed methodology and to assess the accuracy of the conventional and refined semi-analytical approaches for shape variables.

2. Finite element analysis

The finite element analysis of structures subjected to thermal loading will be briefly discussed here. The objective is to present some features that are important to the derivation of the expressions used in the sensitivity analysis, which will be effectively carried out in Section 3. More details of FE procedures used to trace the complete nonlinear equilibrium paths of structures subject to thermal loading, including algorithms to trace the secondary (post-buckling) paths are presented in [8].

In order to include the thermal effects in the analysis of structures it is necessary to modify the relations between stresses ($\boldsymbol{\sigma}$) and strains ($\boldsymbol{\varepsilon}$) to consider the expansion due to temperature changes. For linear elastic materials, the stress–strain–temperature relation can be written as

$$\boldsymbol{\sigma} = \mathbf{C}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_{\text{th}}), \quad (1)$$

where $\mathbf{C}$ is the temperature-dependent elastic constitutive matrix and $\boldsymbol{\varepsilon}_{\text{th}}$ are thermal strains of a solid whose expansion is not constrained.

The thermal strains of a 3D isotropic solid can be computed from

$$\boldsymbol{\varepsilon}_{\text{th}} = \boldsymbol{\alpha}\Delta T \quad \text{with } \boldsymbol{\alpha}^{t} = [\alpha \quad \alpha \quad \alpha \quad 0 \quad 0 \quad 0], \quad (2)$$

where α is the thermal expansion coefficient and ΔT is the temperature variation. Similar expressions arise for plane stress, plane strain, and axisymmetric solid models.