Available online at www.sciencedirect.com
ScienceDirect
Mathematical and Computer Modelling 47 (2008) 1134–1149
MATHEMATICAL AND COMPUTER MODELLING
www.elsevier.com/locate/mcm

Parameter uncertainty, sensitivity analysis and prediction error in a water-balance hydrological model

Kurt K. Benke[a,*], Kim E. Lowell[a,b], Andrew J. Hamilton[a,c]

[a] Primary Industries Research Victoria – Parkville Centre, Department of Primary Industries, PO Box 4166, Parkville, Victoria 3052, Australia
[b] CRC for Spatial Information, The University of Melbourne, 723 Swanston Street, Ground floor, Carlton, Victoria 3052, Australia
[c] School of Resource Management, Faculty of Land and Food Resources, The University of Melbourne, 500 Yarra Boulevard, Richmond, Victoria 3121, Australia

Received 20 April 2007; accepted 16 May 2007

Abstract

Analysis of uncertainty is often neglected in the evaluation of complex systems models, such as computational models used in hydrology or ecology. Prediction uncertainty arises from a variety of sources, such as input error, calibration accuracy, parameter sensitivity and parameter uncertainty. In this study, various computational approaches were investigated for analysing the impact of parameter uncertainty on predictions of streamflow for a water-balance hydrological model used in eastern Australia. The parameters and associated equations which had greatest impact on model output were determined by combining differential error analysis and Monte Carlo simulation with stochastic and deterministic sensitivity analysis. This integrated approach aids in the identification of insignificant or redundant parameters and provides support for further simplifications in the mathematical structure underlying the model. Parameter uncertainty was represented by a probability distribution and simulation experiments revealed that the shape (skewness) of the distribution had a significant effect on model output uncertainty. More specifically, increasing negative skewness of the parameter distribution correlated with decreasing width of the model output confidence interval (i.e. resulting in less uncertainty). For skewed distributions, characterisation of uncertainty is more accurate using the confidence interval from the cumulative distribution rather than using variance. The analytic approach also identified the key parameters and the non-linear flux equation most influential in affecting model output uncertainty.

Keywords: Complex systems; Error propagation; Hydrological model; Monte Carlo simulation; Risk; Sensitivity analysis; Uncertainty; 2C; 2CSalt

1. Introduction

Hydrological models have been widely used in the past to provide catchment management with information on the interaction of water, energy and vegetation processes distributed over space and time [1]. Computational models can be used to quantify surface and groundwater contributions to streamflow and salt export at catchment scale, and have particular importance with respect to the effect of changes in land-use. For example, the impact of land-usage on salt and water yield would require the evaluation of tree-planting strategies, the consequent effect on bio-diversity, and

* Corresponding author.
E-mail address: kurt.benke@dpi.vic.gov.au (K.K. Benke).

0895-7177/$ - see front matter
doi:10.1016/j.mcm.2007.05.017

optimisation of land-use with respect to impact on stream salinity. In particular, an imbalance in the proportion of land devoted to urban, farming and forestry planning could dramatically reduce water available to streamflow and storage in a catchment.

A typical hydrological model consists of a large number of coupled equations describing the direction of water flow, including surface and sub-surface flows, providing predictions of monthly and annual streamflow or salt deposition. Additional inputs represent the spatial mosaic of climate, soil type, topography and land use [2]. Temporal inputs include estimates of surface runoff, sub-surface lateral flow, recharge, and potential evaporation. The computational procedure may estimate the partitioning between surface, lateral and groundwater pathways in the catchment and be applied over daily, monthly or yearly increments.

Historically, most hydrological models have been deterministic in that model parameters and inputs are represented by single values or point estimates — e.g. 2C or 2CSalt [3], BC2C [4] and other coupled salt and water-balance models [5]. More recently, reviews have been published on the role of calibration and uncertainty in the modelling process [1,6]. With respect to the specific aim of calibration and parameter estimation, uncertainty has been addressed by proposing various regression and probabilistic approaches, such as Generalised Likelihood Uncertainty Estimation (GLUE) [7], Markov Chain Monte Carlo (MCMC) [8], and also MCMC/Bayesian inference approaches, such as MCMC Global Sensitivity Analysis (MCMC-GSA) [9] and Bayesian Approach to Total Error Analysis (BATEA) [10,11].

In general, hydrological models incorporate many parameters (some statistical and some with physical significance), most of which require measurements from resource-intensive field exercises which are used to calibrate the model by statistical methods, such as least-squares regression analysis or the approaches cited in the previous paragraph. In some cases, parameters with physical significance may be adjusted interactively during calibration. Irrespective of the method used for parameter estimation, one often has little sense of which parameters have the most influence on model output. Indeed, some parameters may have such little impact that they could be easily ignored, leading to simplification of the mathematical structure of the model. A primary concern in this study was to explore the sensitivity of predictions to parameter variability in order to establish their relative importance for accurate calibration.

By including uncertainty in model parameters, rather than using point estimates, more information is available to the catchment manager with respect to prediction error. In a fundamental sense, uncertainty associated with model output may be represented as a probability distribution or as a specific statistical quantity, such as the 95th percentile result from the cumulative probability distribution (i.e. what is the annual streamflow prediction with a 95% probability?). By introducing notions of confidence and probability, this approach provides more information than a single point estimate and informs policy developers about the degree of risk associated with particular actions (Fig. 1).

Kuczera and Parent [8] studied MCMC/Bayesian approaches to calibration uncertainty and reported a case study for the CATPRO salinity model. During calibration, they observed a bimodal histogram for one parameter, which they attributed to non-stationarity in the time-series input data. In general, however, there has been very little work reported on the shape of parameter distributions and the effect on prediction uncertainty. One objective of the current investigation was to examine the effect of parameter uncertainty on prediction uncertainty in a prototypical hydrological model (a water-balance model), where the parameters have already been assigned. The fixed parameters were subjected to systematic and random perturbations and the effect observed at the model output. The parameters were represented by distributions and the effect on prediction error and uncertainty explored by changing shape (skewness) under a variety of conditions using Monte Carlo simulation.

Another aim of the study was to present catchment managers and field hydrologists with straightforward computational strategies for the assessment of prediction uncertainty in a hydrological model. As a decision aid, this would benefit in the estimation of error, precision and confidence in the model predictions. In particular, isolation of those parameters with most effect on output would support the design and calibration of field experiments (noting cost reductions possible due to avoiding unnecessary measurement and excessive computational complexity). Whilst these investigations were carried out on the 2C hydrological model (a variant of the 2CSalt model), insights into how to deal with uncertainty are applicable to models of similar structure [3–5]. More specific aims of the investigation were (i) to investigate the transfer of uncertainty from designated key parameters to the model output by means of appropriate metrics, (ii) to examine the influence of the shape (skewness) in the parameter distributions on prediction error, and (iii) to conduct sensitivity analysis using both point estimates and parameter distributions.