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Shape sensitivity analysis of composites in contact using the boundary element method

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ABSTRACT

This paper presents the application of the boundary element method to the shape sensitivity analysis of two-dimensional composite structures in contact. A directly differentiated form of boundary integral equation with respect to geometric design variables is used to calculate shape design sensitivities for anisotropic materials with frictionless contact. The selected design variables are the coordinates of the boundary points either in the contact or non-contact area. Three example problems with anisotropic material properties are presented to validate the applications of this formulation.

1. Introduction

Shape sensitivity analysis, that is the calculation of quantitative information on how the response of a structure is affected by changes in the variables that define its shape, is a fundamental requirement for shape optimization. Shape optimization is an important area of current development in mechanical and structural design. Computerized procedures using optimization algorithms can iteratively determine the optimum shape of a component while satisfying some objectives, without at the same time violating the design constraints. The boundary element method being a surface-oriented technique is well suited for shape optimization problems [1–5].

In the last two decades important advancements have been made in the analysis of contact problems using the finite element or boundary element methods. The latter seems to have proved advantageous in treating the contact between linear elastic solids [6–8]. The contact surface design is usually the first requirement to reduce the stress peaks. Therefore, various efforts have been made to produce optimal designs which increase the performance and reliability of the structure in contact environments [9–13]. The effect of material properties should next be considered in conjunction with the shape optimization to obtain the required performance of the component. However, in this field of research the analysis has been mostly concentrated on isotropic materials.

The application of composites in aerospace, automobile, civil and marine industries is well established today due to the known benefits such as high specific stiffness or strength and the material's tailoring facilities for creating high-performance structures. An understanding of the interactions between the composite material components and their optimum contact surface design can further enhance their potential applications.

The objective of this work is directed towards the shape sensitivity analysis of two-dimensional anisotropic structures in frictionless contact. This study continues the previous works of the author on the shape optimization of anisotropic structures using the boundary element method [1–3], where the effect of material properties on the optimum shape design of structures was investigated.

In Ref. [3], a directly differentiated form of the BIE, with respect to boundary point coordinates, was used to calculate stress and displacement derivatives for two-dimensional anisotropic structures. In Ref. [2], the optimal shape design of an anisotropic elastic body of maximum stiffness and minimum weight under specified loadings and using the boundary element method, was obtained. The elastic compliance of the structure was minimized while there were constraints on the maximum stress and weight of the structure. The objective of the work in Ref. [1] was directed towards the optimal positioning of features in anisotropic structures for maximum stiffness while the weight remains unchanged. The elastic compliance was minimized while there were constraints on the maximum stress and the geometry of the structure. To the author's knowledge, no other publications are available on the shape optimization of composite materials using the boundary elements.

Here, the design sensitivity analysis of composite structures in contact has been carried out by direct differentiation of the

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structural response rather than using the finite difference method. The design variables are taken as the coordinates of some nodes on the boundaries of either body which is in contact. The selection of the boundary points as the design variables is more general than selecting simple geometrical variables such as radii, etc. The advantage of the proposed method is that it can be applied to any geometry, not necessarily regular shapes. However, when entire segments of the boundary or domain are governed by a single variable such as radius, the relevant velocity terms are applied together in the sensitivity analysis with respect to that variable [1]. The formulation obtained in the present study may be employed in conjunction with any numerical optimization algorithm for the shape optimization of anisotropic components in contact.

2. Constitutive equations for plane anisotropic elasticity

The stress–strain relations for a two-dimensional homogeneous, anisotropic elastic body in plane stress is

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} \tag{1}$$

where σ_{jk} and ε_{jk} $(j,\ k=1,\ 2)$ are the stresses and strains, respectively [14]. The coefficients a_{mn} are the elastic compliances of the material. These compliances can be written in terms of engineering constants as

$$\begin{aligned} &a_{11}=\frac{1}{E_1},\ a_{22}=\frac{1}{E_2},\quad a_{12}=-\frac{v_{12}}{E_1}=-\frac{v_{21}}{E_2} \\ &a_{66}=\frac{1}{G_{12}},\ a_{16}=\frac{\eta_{12,1}}{E_1}=\frac{\eta_{1,12}}{G_{12}},\quad a_{26}=\frac{\eta_{12,2}}{E_2}=\frac{\eta_{2,12}}{G_{12}} \end{aligned} \tag{2}$$

where E_k is Young's modulus in the x_k direction, G_{12} is the shear modulus in the x_1–x_2 plane and v_{jk} is Poisson's ratio. The quantities $\eta_{jk,1}$ and $\eta_{1,jk}$ are referred to by Lehknitskii [15] as the coefficients of mutual influence of the first and second kinds, respectively. For the plane strain case, Eq. (1) remains applicable, provided that a_{jk} is replaced by b_{jk}:

$$\begin{aligned} &b_{jk}=a_{jk}-\frac{a_{j3}a_{k3}}{a_{33}},\ j,\ k=1,\ 2,\ 6 \\ &a_{j3}=\ -\frac{v_{j3}}{E_j},\ a_{33}=\frac{1}{E_3},\quad a_{63}=\frac{\eta_{12,3}}{E_3}=\frac{\eta_{3,12}}{G_{12}} \end{aligned} \tag{3}$$

where the index 3 refers to the x_3 direction. For especially orthotropic materials, $a_{16}=a_{26}=a_{63}=0$ [15].

The compatibility equation of strains is

$$\frac{\partial^2\varepsilon_{11}}{\partial x_2^2}+\frac{\partial^2\varepsilon_{22}}{\partial x_1^2}=2\frac{\partial^2\varepsilon_{12}}{\partial x_1\partial x_2} \tag{4}$$

and equilibrium is satisfied by taking stresses in terms of derivatives of the Airy stress function $\phi(x_1,\ x_2)$ as

$$\sigma_{11}=\frac{\partial^2\phi}{\partial x_2^2},\ \sigma_{22}=\frac{\partial^2\phi}{\partial x_1^2},\quad \sigma_{12}=-\frac{\partial^2\phi}{\partial x_1\partial x_2} \tag{5}$$

Combining Eqs. (1), (4) and (5), the governing equation for the two-dimensional problem of anisotropic elasticity can be obtained as

$$\begin{aligned} &a_{22}\frac{\partial^4\phi}{\partial x_1^4}-2a_{26}\partial\frac{\partial^4\phi}{\partial x_1^3\partial x_2}+(2a_{12}+a_{66})\frac{\partial^4\phi}{\partial x_1^2\partial x_2^2}-2a_{16}\frac{\partial^4\phi}{\partial x_1\partial x_2^3} \\ &\quad +a_{66}\frac{\partial^4\phi}{\partial x_2^4}=0 \end{aligned} \tag{6}$$

By introducing the operator D_s $(s=1,\ 4)$ as

$$D_s=\frac{\partial}{\partial x_2}-\mu_s\frac{\partial}{\partial x_1} \tag{7}$$

Eq. (6) becomes

$$D_1D_2D_3D_4(\phi)=0 \tag{8}$$

and μ_s are the four roots of the characteristic equation

$$[a_{22}-2\mu a_{26}+(2a_{12}+a_{66})\mu^2-2a_{16}\mu^3+a_{11}\mu^4]\frac{\mathrm{d}^4\phi}{\mathrm{d}z^4}=0 \tag{9}$$

To have a solution for the stress function, the term in square brackets must be zero. Lehknitskii [15] has shown that, for an anisotropic material, these roots are distinct and must be purely imaginary or complex and they may be denoted by

$$\mu_1=\alpha_1+\mathrm{i}\beta_1,\ \mu_1=\alpha_2+\mathrm{i}\beta_2,\quad \mu_3=\bar{\mu}_1,\ \mu_4=\bar{\mu}_2 \tag{10}$$

where α_j and β_j $(j=1,\ 2)$ are real constants, $\mathrm{i}=\sqrt{-1}$ and the overbar represents the complex conjugate. Therefore, the stresses and displacements in an anisotropic body can be expressed in terms of the complex coordinates (z_j)

$$z_j=x_1+\mu_j x_2,\ j=1,\ 2 \tag{11}$$

and their complex conjugates.

3. Review of the boundary element method for anisotropic materials in contact

The analytical formulation of the direct boundary integral equation (BIE) for plane anisotropic elasticity may be developed by following the same steps as in the isotropic case [16–18].

The boundary element method is based on the unit load solutions in an infinite body, known as the fundamental solutions, used with the reciprocal work theorem and appropriate limit operations. The boundary integral equation (BIE) of the BEM for anisotropic materials is an integral constraint equation relating boundary tractions (t_j) and boundary displacements (u_j) and it may be written as

$$C_{jk}u_j(P)+\int_s T_{jk}(P,Q)u_j(Q)\,\mathrm{d}s(Q)=\int_s U_{jk}(P,Q)t_j(Q)\,\mathrm{d}s(Q),\ j,\ k=1,2 \tag{12}$$

$P(\zeta_1,\zeta_2)$ and $Q(x_1,x_2)$ are the field and load points, respectively. $U_{jk}(P,Q)$ and $T_{jk}(P,Q)$ are the fundamental solutions that represent the displacements and tractions, respectively, in the x_k direction at Q because of a unit load in the x_j direction at P in an infinite body. The constant C_{jk} depends on the local geometry of the boundary at P, whether it lies on a smooth surface or a sharp corner. In terms of the generalized complex variables

$$\begin{aligned} &z_1=(x_1-\zeta_1)+\mu_1(x_2-\zeta_2) \\ &z_2=(x_1-\zeta_1)+\mu_2(x_2-\zeta_2) \end{aligned} \tag{13}$$

the fundamental solutions for displacements and tractions, respectively, can be written as

$$\begin{aligned} &U_{jk}=2\,\mathrm{Re}[r_{k1}A_{j1}\ \ln(z_1)+r_{k2}A_{j2}\ln(z_2)] \\ &T_{j1}=2n_1\,\mathrm{Re}[\mu_1^2A_{j1}/z_1+\mu_2^2A_{j2}/z_2]-2n_2\,\mathrm{Re}[\mu_1A_{j1}/z_1+\mu_2A_{j2}/z_2] \\ &T_{j2}=\ -2n_1\,\mathrm{Re}[\mu_1A_{j1}/z_1+\mu_2A_{j2}/z_2]+2n_2\,\mathrm{Re}[A_{j1}/z_1+A_{j2}/z_2] \end{aligned} \tag{14}$$

where n_j are the unit outward normal components at Q with respect to the x_1–x_2 coordinate system. The constants r_{kj} are

$$\begin{aligned} &r_{1j}=a_{11}\mu_j^2+a_{12}-a_{16}\mu_j \\ &r_{2j}=a_{12}\mu_j+a_{22}/\mu_j-a_{26} \end{aligned} \tag{15}$$