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## Buckling sensitivity analysis of cracked thin plates under membrane tension or compression loading

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#### ABSTRACT

Thin-walled structural components such as plates and shells are commonly used in practical applications such as aerospace, naval, nuclear power plant, pressure vessels, mechanical and civil engineering structures and so on, and the safety assessment of such structures must carefully consider all the phenomena which can decrease the bearing capacity of such elements. Among them, the presence of cracks in thin-walled structures can heavily affects their safety factor with respect to the more common modes of failure such as buckling or fracture. For very thin plate, buckling collapse under compression or even under tension, apart fracture or plastic failure in this last case, can easily take place: the presence of flaws such as through-the-thickness cracks can sensibly modify such ultimate loads. In the paper the effects of cracks' length and orientation on the buckling loads of rectangular elastic thin-plates - characterised by different boundary conditions and by various Poisson's ratio – under tension and compression, is considered. For tensioned flawed plates a fracture-buckling and a plastic-buckling collapse maps are obtained. After a short explanation of the buckling phenomena in plates, several FE numerical parametric analyses results are presented in terms of critical load multiplier in compression or in tension in cracked plates. The obtained results are discussed and some interesting and useful conclusions regarding the sensitivity to cracks' presence of buckling loads of thin plates under compression or tension (or fracture in this last case) are explained. The interesting case of tensioned cracked plates is considered by studying the easiest collapse between fracture, plastic flow and buckling: in such cases some failure-type maps are finally determined.

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#### 1. Introduction

The presence of cracks in thin-walled structures, such as plates and shells used in different engineering applications, is a quite common situation. Since flaws can heavily affects the safety of such structures with respect to the more common modes of failure such as buckling, plastic flow or fracture, failure due to cracks can very often precedes the most common strength collapse. When the plate's thickness is sufficiently small with respect to others plate's sizes, buckling collapse under compression or even under tension (it usually occurs as a local buckling phenomena around imperfections, due to compression which develops transversally loading direction), if fracture has not yet occurred, can take place: the presence of flaws such as through-the-thickness cracks can sensibly modify such ultimate loads.

Thin-walled high-strength panels have several application in many engineering fields such as aerospace, naval, mechanical, power industries, mechanical, civil structures, etc.; in order to

reduce economic costs and to design high performance structures, the evaluation of the safety levels in such a structural components is very important and for these reasons several investigations have been carried out in order to determine the buckling load, responsible of the failure phenomena in damaged or undamaged plates under compression (Wang et al., 1994; Matsunaga, 1997; Byklum and Amdahl, 2002; Bert and Devarakonda, 2003; Paik et al., 2005) or under tension (Markström and Storåkers, 1980; Sih and Lee, 1986; Shaw and Huang, 1990; Shimizu and Enomoto, 1991; Nageswara, 1992; Riks et al., 1992; Friedl et al., 2000; Guz et al., 2001; Brighenti, 2005; Paik et al., 2005).

The effects of imperfections on the buckling behaviour of plates and shells have been studied in some research; in particular the influence of holes (Shimizu and Enomoto, 1991) and cracks on the buckling load in compressed or tensioned homogeneous (Markström and Storåkers, 1980; Sih and Lee, 1986; Shaw and Huang, 1990; Riks et al., 1992; Estekanchi and Vafai, 1999; Guz et al., 2001; Vafai and Estekanchi, 1999; Vafai et al., 2002; Dyshel, 2002; Brighenti, 2005; Paik et al., 2005) or composite plates (Barut et al., 1997) and shells (Alinia et al., 2007) have been carried out. The buckling load multiplier in compressed or tensioned plates is heavily affected by the presence of a crack, and particularly the

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#### Nomenclature

a crack half length

 $a^* = a/W$  dimensionless crack length

 $a_0, \ldots, a_3, b_1, \ldots, b_3, c_1, \ldots, c_3$  coefficients of the polynomial interpolation of the buckling stress multipliers for cracked plates

D plate's flexural rigidity

E, G plate's Young and shear modulus, respectively

E' = E or  $E' = E/(1 - v^2)$  reduced elastic modulus in plane stress or plain strain, respectively

 $F_{col,1}$ ,  $F_{col,2}$  collapse functions that identify buckling failure with respect to fracture ( $F_{col,1} > 0$ ) or with respect to plastic failure ( $F_{col,2} > 0$ ) of tensioned cracked plates

k minimum value attained by a function of the dimensionless parameter  $W^* = W/L$ 

 $K_{eq}$ ,  $K_{\rm I}$ ,  $K_{\rm II}$ ,  $K_{\rm III}$  equivalent, mode I, mode II and mode III Stress-Intensity Factor in mixed mode fracture, respectively

 $K_{IC}$  fracture toughness

L, W plate's half length and width, respectively

 $N_x$ ,  $N_y$ ,  $N_{xy}$  membrane and shearing forces (per unit length of the plate) in the corresponding directions, respectively

t plate's thickness

 $t^* = t/W$  dimensionless plate's thickness

w(x, y) plate's elastic surface (transversal displacements)

 $W^* = W/L$  dimensionless plate's width (plate's aspect ratio)

#### Greek letters

 $\beta = K_{IC}/\sigma_{E,c}$  ratio between the fracture toughness and the compressive critical stress for compressed uncracked plates

λ<sup>-</sup>, λ<sup>+</sup> buckling stress multipliers in compression and in tension divided by the buckling stress of an uncracked plate, respectively

 $\tilde{\lambda}^-, \tilde{\lambda}^+$  Approximate buckling stress multipliers in compression and in tension divided by the buckling stress of an uncracked plate, respectively, obtained by polynomial fitting of the FE results

 $\lambda^{*-}$ ,  $\lambda^{*+}$  buckling stress multipliers in compression and in tension divided by the elastic modulus, respectively Poisson's ratio

 $\theta,\bar{\theta}$  crack orientation angle and transition angle, respectively

 $\sigma_0^-, \sigma_0^+$  actual applied buckling compressive or tensile stress, respectively

 $\sigma_{c}(a^{*}, \theta, v, K_{IC})$  fracture collapse stress for a tensioned cracked plate

 $\sigma_{E,c}$  Euler buckling (uniform) stress in compression for uncracked plates

 $\sigma_p(a^*,\,\theta,\,\sigma_y)$  plastic collapse stress for a tensioned cracked plate

 $\sigma_v$  material's yield stress

effects of the relative crack length and orientation must be studied carefully as well as the boundary condition of the structures being examined in order to accurately assess their safety factor. Moreover it should be considered that buckling under tension, that seems to be an unrealistic phenomena, easily appears as a local phenomena characterised by complex wrinkling deflection patterns in compressed regions which develops around defects such as cracks or holes, (Markström and Storåkers, 1980; Sih and Lee, 1986; Shaw and Huang, 1990; Shimizu and Enomoto, 1991; Riks et al., 1992;

Estekanchi and Vafai, 1999; Guz et al., 2001; Vafai and Estekanchi, 1999; Vafai et al., 2002; Dyshel, 2002; Brighenti, 2005; Paik et al., 2005)

In the present paper a systematic study of the effects of cracks' length and orientation on the buckling loads of rectangular elastic thin-plates – characterised by different boundary conditions and by various Poisson's ratio – under tension or compression, is considered.

Firstly a short explanation of the buckling phenomena in plates is presented and the results concerning several numerical parametric analyses, carried out by performing Finite Element Method (FEM) linear buckling analyses, are presented in terms of the critical load multiplier in compression or in tension. In particular, the buckling critical load multiplier is determined for different crack length and orientation with respect to the load direction, Poisson's coefficient of the plate's material (from low values up to nearly incompressible materials) and by varying the plate's boundary conditions.

The obtained results are graphically summarised in some figures and tables, while some useful interpolations expressions of the obtained results are finally given for practical cases, characterised by geometrical parameters ranging in the considered intervals. For tensioned cracked plates a fracture-buckling and a plastic-buckling collapse maps are obtained in order to identify the easiest collapse mode of such structures. Finally the presented results are discussed and some interesting and useful conclusions regarding the sensitivity on cracked plates compression or tension buckling failure of the above mentioned parameters, are drawn.

#### 2. Formulation of the problem

When the safety of thin-walled structural elements has to be evaluated, the membrane effects must be absolutely considered in the governing equation: second order geometrical effects must be taken into account (Von Kármán plate's theory, see Timoshenko and Gere, 1961) in the fourth-order partial differential equation which describes the plate's deflection w(x, y) (elastic surface):

$$\nabla^{4}w(x,y) = \frac{1}{D} \cdot (N_{x} \cdot w,_{xx}(x,y) + 2 \cdot N_{xy} \cdot w,_{xy}(x,y) + N_{y} \cdot w,_{yy}(x,y))$$
(1)

with the appropriate boundary conditions; in Eq. (1) the notations  $(\bullet)_{,ij}$  indicates the partial derivatives with respect to the geometric variables  $i=x,\ y,\ j=x,\ y$  (the medium plane of the plate belongs to the  $x,\ y$  plane), the operator  $\nabla^4$  stands for  $\nabla^4 \bullet = (\bullet)_{,xxxx} + 2(\bullet)_{,xxyy} + (\bullet)_{,yyyy} = \frac{\partial \bullet}{\partial x^4} + 2\frac{\partial \bullet}{\partial x^2y^2} + \frac{\partial \bullet}{\partial y^4}$ , and  $N_x,\ N_y,\ N_{xy}$  are membrane forces (per unit plate's length) in the corresponding directions, while  $D=E\cdot t^3/12(1-\nu^2)$  is the plate's flexural rigidity and t is the plate's thickness, respectively. In the considered load configuration  $N_x = \sigma_0 \cdot t,\ N_y = N_{xy} = 0$  and Eq. (1) simplify as  $\nabla^4 w(x,y) - (\sigma_0 \cdot t/D) \cdot w_{,xx}(x,y) = 0$ .

The solution of Eq. (1) can be obtained in a closed-form only for some simple plate's geometry, boundary conditions and membrane load distribution; as an example in the case of a simply supported compressed uncracked rectangular plate, the buckling stress  $\sigma_{E,C}$  (the subscripts stands for Euler stress in a compressed plate) can be expressed as (Timoshenko and Gere, 1961):

$$\sigma_{E,c} = k \cdot \frac{\pi^2 \cdot D}{4W^2 \cdot t} \tag{2}$$

where k is the minimum value attained by a function  $k = k(W^*)$  of the dimensionless parameter  $W^* = W/L$  (Fig. 1).

The buckling load multipliers  $\lambda^-$  and  $\lambda^+$  in compressed and in tensioned generic cracked plates, respectively, can be conveniently expressed in dimensionless form as the ratio between the actual applied buckling stress,  $\sigma_0^-$  or  $\sigma_0^+$ , and the corresponding buckling

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