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# Sensitivity analysis of frequency response functions of composite sandwich plates containing viscoelastic layers

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ABSTRACT

In the scope of structural dynamics, sensitivity analysis is a very useful tool in a number of numerical procedures such as parameter identification, model updating, optimal design and uncertainty propagation. In this paper the formulation of first-order sensitivity analysis of complex frequency response functions (FRFs) is developed for composite sandwich plates composed by a combination of fiber-reinforced and elastomeric viscoelastic layers, in arrangements that are frequently used for the purpose of noise and vibration attenuation. Although sensitivity analysis is a well known numerical technique, the main contribution intended for this study is its extension to viscoelastic structures, which are characterized by frequency- and temperature-dependent material properties and, thus, require particularly adapted analytical and numerical procedures. Due to the fact that finite element discretization has become the most used method for dynamic analysis of complex structures, the sensitivity analysis addressed herein is based on such models, being computed from the analytical derivatives of the FRFs with respect to a set of design parameters, such as fiber orientations and layer thicknesses. Also, a procedure for evaluating the sensitivity of the FRFs with respect to temperature of the viscoelastic material is suggested. After discussion of various theoretical aspects, including a parameterization scheme of the structural matrices with respect to the design variables, first-order response derivatives are calculated for a composite plate with inherent structural damping, and for a composite sandwich plate with a viscoelastic core. The results are compared to those obtained from first-order finite-difference approximations.



## 1. Introduction

Composite materials have been increasingly used in various types of engineering systems, especially in aerospace structures, in which structural components must be designed to withstand harsh static and dynamic loading conditions, with typically high reliability levels. This fact makes indispensable the availability of efficient numerical models capable of predicting various aspects of the structural behavior: static and dynamic responses, noise radiation, damage initiation and propagation, etc. However, the large variety of material properties and structural configurations makes the numerical modeling of composite structures a complex task [1]. This is a reason for which in the last decades, a great deal of effort has been devoted to the development of finite element models for characterizing the mechanical behavior of such materials, accounting for its typical variations of construction features and material properties. Comprehensive studies on this subject have been reported in the monographs by Reddy [1] and Gay et al. [2].

In applications in which dynamic loads are involved, the interest in achieving vibration attenuation becomes of capital importance as vibration amplitudes are directly related to fatigue and, as a result, to structural integrity [3,4]. Among the various techniques for vibration control which have been devised, the so-called *passive techniques* have been incorporated in many industrial systems as they present a number of advantages as compared to *active* strategies such as cost effectiveness, broadband operational effectiveness and inherent stability [4]. Typically, passive vibration control can be achieved by using energy-dissipating materials, such as a class of elastomers.

It is known that fiber-reinforced composite materials present inherent damping mechanisms associated to the viscoelastic behavior of the polymeric matrices and other internal dissipation mechanisms [1,2]. Nonetheless, in a number of cases, such damping may be found to be insufficient to provide the necessary vibration mitigation and must be increased by introducing additional dissipation, which can be done, for example, by applying viscoelastic materials in the form of surface treatments or internal layers [4,5]. This strategy is considered in this paper.

Among the various theories which have been developed for modeling layered composite structures, the so-named *Higher-order*

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*Shear Deformation Theory – HSDT* proposed by Lo et al. [6] has been chosen in the present study. Despite a more involved analytical framework and an increased number of finite-element degrees-of-freedom (d.o.f's), when compared to other simpler theories, the main advantages of the HSDT, which justify such choice, are: (i) it is well adapted to model both thin and thick laminated composite plates; (ii) it accounts for a more complete strain state. As it takes into account transverse shear effects and predicts a parabolic distribution of transverse shear strains, it does not require any correction factor for the distribution of transverse shear strains, such as those required by the *First-order Shear Deformation Theory – FSDT*. Moreover, it accounts for transverse normal strains.

The fact of considering a more complete strain-state is believed to be a relevant fact since the amount of energy dissipation provided by the viscoelastic materials is primarily determined by the level and history of strains within the material [7]. Hence, the more accurate the prediction of strain is, the more accurate the prediction of damping will be.

In the context of the numerical methods for analysis and design of mechanical systems based on finite element models, *sensitivity analysis* enables to evaluate the degree of influence of physical and geometrical parameters on the mechanical behavior; it constitutes an important step in various types of problems such as model updating, analysis of modified structures, optimal design, system identification, system control and uncertainty propagation [8]. According to Murthy and Haftka [9], the optimal design structural systems has a narrow connection with sensitivity analysis, since a significant part of typical optimization algorithms generally perform a large number of evaluations of the system's response for different values of the design variables. Derivatives can be used to approximate the response of modified systems, thus reducing the cost of re-analysis, especially for highly complex industrial structures.

Sensitivity analysis can be performed through two main types of numerical procedures [10]: first by computing the analytical derivatives of the equilibrium equations or equations of motion with respect to the design parameters, when those parameters appear explicitly in such equations. In this case, the derivatives are considered to be exact; second by approximating the derivatives by finite differences, by computing the ratios of the response and parameter variations. Such procedure requires the arbitrary choice of the magnitude of the parameter variation and two evaluations of the system response, for each derivative estimate. Most frequently, sensitivity analysis is limited to the first-order, as computations of higher orders are much more involved and time-consuming.

Several approaches have been developed for performing sensitivity analysis of dynamic responses, as reported in Refs. [10–12]. However, applications to the case of vibration and damping analysis of composite sandwich plates incorporating viscoelastic materials are not numerous, which motivates the study reported herein.

In the remainder, various theoretical aspects are first presented including: (a) a review of the finite element modeling procedure of multilayered plate structures containing unidirectional fiber-reinforced and isotropic viscoelastic layers, with a special parameterization scheme of the structural matrices with respect to the design variables; (b) the inclusion of the viscoelastic effect into the structural matrices using the complex modulus approach, accounting the frequency- and temperature-dependence of the properties of the viscoelastic material; (c) the formulation of the analytical sensitivities of the frequency response functions with respect to the physical and geometrical parameters and also with respect to the temperature of the viscoelastic material. For illustration, two numerical examples are presented. For each of them, the FRF sensitivities computed according to the methodology suggested are compared to first-order finite-difference approximations.

## 2. Finite element formulation of laminated composite plates

In this section the formulation of a composite plate finite element, depicted in Fig. 1, is summarized based on the original developments made by Lo et al. [6] and Chee et al. [13].

Where $z_k$, $h_k$ and $\theta_k$ indicate, respectively, the thickness coordinate, the thickness and the fiber orientation angle of the $k$th unidirectional layer.

According to the *Higher-order Shear Deformation Theory* (HSDT), the displacements at an arbitrary point of the element are expressed as follows:

$$\boldsymbol{U}(x,y,z,t) = \boldsymbol{A}(z)\,\boldsymbol{u}(x,y,t) \tag{1}$$

In Eq. (1):

$$\boldsymbol{U}(x,y,z,t) = [u(x,y,z,t)\ v(x,y,z,t)\ w(x,y,z,t)]^T \tag{2.a}$$

$$\boldsymbol{A}(z) = \begin{bmatrix} 1 & 0 & 0 & z & 0 & 0 & z^2 & 0 & 0 & z^3 & 0 \\ 0 & 1 & 0 & 0 & z & 0 & 0 & z^2 & 0 & 0 & z^3 \\ 0 & 0 & 1 & 0 & 0 & z & 0 & 0 & z^2 & 0 & 0 \end{bmatrix} \tag{2.b}$$

$$\begin{aligned}\boldsymbol{u}(x,y,t) = \big[&u_0(x,y,t)\,v_0(x,y,t)\,w_0(x,y,t)\,\psi_x(x,y,t)\,\psi_y(x,y,t)\ \psi_z(x,y,t)\cdots \\ &\times\zeta_x(x,y,t)\,\zeta_y(x,y,t)\,\zeta_z(x,y,t)\,\Phi_x(x,y,t)\,\Phi_y(x,y,t)\big]^T\end{aligned} \tag{2.c}$$

where $u(x,y,z,t)$, $v(x,y,z,t)$, and $w(x,y,z,t)$ denote the displacements in directions $x$, $y$ and $z$, respectively. $(u_0, v_0, w_0)$ and $(\psi_x, \psi_y, \psi_z)$ are, respectively, the mid-plane displacements and the cross-section rotations in $x$, $y$ and $z$ directions. The terms $\zeta_x$, $\zeta_y$, $\zeta_z$, $\Phi_x$ and $\Phi_y$, can be regarded as higher order rotations, lacking a clear geometrical interpretation [13]. From Eq. (1), it can be seen that the displacement approximation in the thickness direction $z$ is made separately from the two other directions, in a procedure which is similar to the traditional separation of variables.

The usual strain–displacement relations are used and the resulting strains are separated in bending and transverse shear strains, $\varepsilon_b$ and $\varepsilon_s$, respectively, as follows:

$$\begin{aligned}\boldsymbol{\varepsilon}_b(x,y,z,t) &= \left[\boldsymbol{D}_0 + z\boldsymbol{D}_1 + z^2\boldsymbol{D}_2 + z^3\boldsymbol{D}_3\right]\boldsymbol{u}(x,y,t) \\ &= \boldsymbol{D}_b(z)\boldsymbol{u}(x,y,t)\end{aligned} \tag{3.a}$$

$$\boldsymbol{\varepsilon}_s(x,y,z,t) = \left[\boldsymbol{D}_4 + z\boldsymbol{D}_5 + z^2\boldsymbol{D}_6\right]\boldsymbol{u}(x,y,t) = \boldsymbol{D}_s(z)\boldsymbol{u}(x,y,t) \tag{3.b}$$

where $\boldsymbol{\varepsilon}_b(x,y,z,t) = [\varepsilon_{xx}\ \varepsilon_{yy}\ \varepsilon_{zz}\ \gamma_{xy}]^T$ and $\boldsymbol{\varepsilon}_s(x,y,z,t) = [\gamma_{yz}\ \gamma_{zx}]^T$. $\varepsilon_{xx} = \partial u/\partial x$, $\varepsilon_{yy} = \partial v/\partial y$, $\varepsilon_{zz} = \partial w/\partial z$, $\gamma_{xy} = (\partial u/\partial y + \partial v/\partial x)$, $\gamma_{yz} = (\partial v/\partial z + \partial w/\partial y)$ and $\gamma_{zx} = (\partial u/\partial z + \partial w/\partial x)$. Matrices $\boldsymbol{D}_i$ $(i = 0,\ldots,6)$ are formed by differential operators appearing in the strain–displacement relations, as detailed in Ref. [1].

Discretization of the displacement variables is made by using appropriate interpolation functions. Hence, for the eight-node rectangular plate element, the 11 mechanical variables included in vector $\boldsymbol{u}(x,y,t)$ are interpolated from their corresponding 88 nodal values through the following relation:

$$\boldsymbol{u}(\xi,\eta,t) = \boldsymbol{N}(\xi,\eta)\,\boldsymbol{u}(t) \tag{4}$$

where $\boldsymbol{u}(t) = \left[\boldsymbol{u}_1^T(t)\ \boldsymbol{u}_2^T(t)\cdots\boldsymbol{u}_8^T(t)\right]^T$, $\boldsymbol{u}_i(t) = \left[u_i\ v_i\ w_i\ \psi_{xi}\ \psi_{yi}\ \psi_{zi}\ \zeta_{xi}\ \zeta_{yi}\ \zeta_{zi}\ \Phi_{xi}\ \Phi_{yi}\right]^T$ ($i$ = 1–8). $\boldsymbol{N}(\xi,\eta)$, of dimensions $11\times 88$, is the matrix formed by the standard serendipity eight-node shape interpolation functions formulated in local coordinates $(\xi,\eta)$, $-1\leqslant\xi\leqslant 1$, $-1\leqslant\eta\leqslant 1$, which are given in Ref. [1].

By associating Eqs. (1–4), the displacement and strain fields are found to be expressed in terms of the nodal values as follows:

$$\boldsymbol{U}(x,y,z,t) = \boldsymbol{A}(z)\boldsymbol{N}(\xi,\eta)\,\boldsymbol{u}(t) \tag{5}$$

$$\boldsymbol{\varepsilon}_b(x,y,z,t) = \boldsymbol{D}_b(z)\boldsymbol{N}(\xi,\eta)\,\boldsymbol{u}(t) = \boldsymbol{B}_b(\xi,\eta,z)\,\boldsymbol{u}(t) \tag{6.a}$$

$$\boldsymbol{\varepsilon}_s(x,y,z,t) = \boldsymbol{D}_s(z)\boldsymbol{N}(\xi,\eta)\,\boldsymbol{u}(t) = \boldsymbol{B}_s(\xi,\eta,z)\,\boldsymbol{u}(t) \tag{6.b}$$