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Phase noise sensitivity analysis of lattice constellation

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Abstract

Based on the assumption of large number of constellation points and high signal-to-noise ratio (SNR), phase noise sensitivity of lattice constellation is analyzed. The upper bound of symbol error rate (SER) in additive white Gaussian noise (AWGN) channel is derived from pairwise error probability. For small phase noise, phase noise channel is transformed to AWGN channel. With the aid of Wiener model, the obtained upper bound can be extended to phase noise channel. The proposed upper bound can be used as performance criterion to analyze the sensitivity of phase noise in multi-dimensional lattice constellation. Simulation results show that with the same normalized spectral efficiency, higher dimensional lattice constellations are more sensitive than lower ones in phase noise channel. It is also shown that with the same dimension of constellation, larger normalized spectral efficiency means more performance loss in phase noise channel.

Keywords lattice constellation, symbol error rate (SER), phase noise

1 Introduction

With the development of communication technologies, high data rate is required urgently, which implies high spectral efficiency due to limited frequency resources. Besides multilevel modulation, lattice constellations are commonly accepted as good methods for transmission with high spectral efficiency [1]. Lattice constellation is modified from infinite lattice, and a lattice is a discrete set of points in real number field. Furthermore, the linearity and highly symmetrical geometry structure of lattice usually simplify the decoding task, such as sphere decoding [2–3] that utilizes the symmetrical structure of lattice and approaches the performance of maximum likelihood with polynomial decoding complexity. Consequently, lattice becomes a research focus in communication field.

Sphere lower bound has been used in Ref. [4] as a benchmark for comparing multi-dimensional constellations in the block-fading channel. The application and approximation of sphere lower bound has been enumerated in Ref. [4]. A new signal model has been introduced in Ref. [5]. Based on the signal model, the exact SER using polar coordinates has been computed in 2-demensional (2-D) constellation. However, to the best of the authors' knowledge, performance analysis of multi-dimensional lattice constellation in phase noise channel has never been addressed.

Phase noise is one of the primary factors that limit the performance of many communication systems. Phase noise is mainly caused by oscillator instability [6–7]. Sensitivity of phase noise is also the measure standard for different communication systems. Consequently, sensitivity analysis of phase noise for multi-dimensional lattice constellation is meaningful. Generally speaking, phase noise is usually modeled as Gaussian model or Wiener model [8]. In this contribution, Wiener model is employed to model the phase noise, and transmitted vector sets are modified from infinitely multi-dimensional lattice. Wiener model signifies the correlation between the phase noise per nearby 2-D component of a lattice constellation.

In this contribution, SER upper bound of lattice constellation in Gaussian channel is derived based on the same assumption of total number of constellation points and average energy per constellation point as in Ref. [1], and offset vector is considered to minimize the average energy per constellation point. Furthermore, the derivation of the upper bound can be extended to phase noise channel in consideration of SNR degradation, and the obtained upper

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bound is tight for larger number of lattice constellation points and higher SNR. Therefore, this upper bound can be used as a criterion to analyze the sensitivity of phase noise of different dimension lattice constellation.

The remainder of this article is organized as follows. The signal model and basic definitions of lattice are introduced in Sect. 2, where phase noise channel with small phase noise is transformed to Gaussian channel by approximation of one-order maclaurin polynomial. The SER upper bound of lattice constellation in Gaussian channel is derived in Sect. 3. Sect. 4 shows the simulation results and conclusions are presented in Sect. 5.

2 Definitions of lattice and signal model

2.1 Definition of lattice

The basic definition of lattice will be reviewed [9–11]. Let $\{\boldsymbol{v}_1,\boldsymbol{v}_2,...,\boldsymbol{v}_m\}$ be a linearly independent set of vectors in $\mathbb{R}^n$ (so that $m \leqslant n$). The set of points $\Lambda=\left\{\boldsymbol{x}=\sum_{i=1}^{m}\lambda_i\boldsymbol{v}_i,\ \lambda_i\in\mathbb{Z}\right\}$ is called a lattice of dimension m, and $\{\boldsymbol{v}_1,\boldsymbol{v}_2,...,\boldsymbol{v}_m\}$ is called a basis of the lattice.

The matrix $\boldsymbol{A}=\begin{pmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & & \vdots \\ v_{m1} & v_{m2} & \cdots & v_{mn} \end{pmatrix}$ is called a generator matrix for the lattice. More concisely, the lattice can be defined by its generator matrix as $\Lambda=\{\boldsymbol{x}=\boldsymbol{\lambda}\boldsymbol{A},\ \boldsymbol{\lambda}\in\mathbb{Z}^m\}$. The Gram matrix for a lattice is defined as the matrix $\boldsymbol{G}=\boldsymbol{A}\boldsymbol{A}^{\mathrm{T}}$, where $(\bullet)^{\mathrm{T}}$ denotes transposition. In following sections, full-rank lattice is considered, that is, $m=n$, and $\boldsymbol{A}$ is a square matrix.

The determinant of a lattice is defined to be the determinant of the Gram matrix $\boldsymbol{G}$. For full-rank lattices, the square root of the determinant is the fundamental volume of the lattice.

Kissing number of a lattice is the number of spheres that touch one sphere in the sphere packing problem [11], i.e., the number of points nearest to one lattice point, where the 'nearest' means the minimum Euclidean distance of a lattice. For a lattice, the kissing number is the same for every lattice point.

2.2 Signal model

The transmitted vector is polluted by both phase noise and AWGN, and the signal model is denoted by

$$\tilde{\boldsymbol{y}}^{\mathrm{T}}=\boldsymbol{H}\tilde{\boldsymbol{x}}^{\mathrm{T}}+\boldsymbol{n}^{\mathrm{T}} \tag{1}$$

where $\boldsymbol{H}=\mathrm{diag}(\boldsymbol{h})\in\mathbb{C}^{N\times N}$, with $\boldsymbol{h}=(\mathrm{e}^{\mathrm{j}\theta_1},\mathrm{e}^{\mathrm{j}\theta_2},......,\mathrm{e}^{\mathrm{j}\theta_n})\in\mathbb{C}^N$, is the phase noise diagonal matrix, and $\tilde{\boldsymbol{y}}\in\mathbb{C}^N$ is the N-dimensional complex received signal vector, with $\boldsymbol{y}=(\mathrm{Re}\,y_1,\mathrm{Im}\,y_1,...,\mathrm{Re}\,y_N,\mathrm{Im}\,y_N)\in\mathbb{R}^{2N}$. $\tilde{\boldsymbol{x}}\in\mathbb{C}^N$ is the N-dimensional complex transmitted signal vector, with $\boldsymbol{x}=(\mathrm{Re}\,x_1,\mathrm{Im}\,x_1,...,\mathrm{Re}\,x_N,\mathrm{Im}\,x_N)\in\mathbb{R}^{2N}$, where $\mathrm{Re}\,\bullet$ and $\mathrm{Im}\,\bullet$ denote the real part and imaginary part, respectively. $\boldsymbol{n}\in\mathbb{C}^N$ is the N-dimensional complex Gaussian noise vector.

The real transmitted signal vector $\boldsymbol{x}$ is assumed to belong to a $2N$-dimensional signal constellation $S\in\mathbb{R}^{2N}$, where S is carved from $2N$-dimension infinite lattice $\Lambda=\{\boldsymbol{u}\boldsymbol{A},\ \boldsymbol{u}\in\mathbb{Z}^{2N}\}$ with full rank generator matrix $\boldsymbol{A}\in\mathbb{R}^{2N\times 2N}$ [10]. With the consideration of normalization purpose, fundamental volume is fixed to 1.

The simplest labeling operation can be used for lattice constellation, that is, $S=\{\boldsymbol{u}\boldsymbol{A}+\boldsymbol{v},\ \boldsymbol{u}\in\mathbb{Z}_m^{2N}\}$, where $\mathbb{Z}_m=\{0,1,...,m-1\}$ [9], $\mathrm{lb}\,m$ is the number of bits per dimension and $\boldsymbol{v}$ is the offset vector used to minimize the average transmitted energy of lattice constellation S. Therefore, the rate of such lattice constellation is $R=\mathrm{lb}\,m$ bit per dimension, which is usually referred to as full-rate uncoded transmission and R is the so-called normalized spectral efficiency per dimension.

For the transmitted symbol $\tilde{x}_i$, the received symbol $\tilde{y}_i$ after the phase noise channel is

$$\tilde{y}_i=\tilde{x}_i\mathrm{e}^{\mathrm{j}\theta_i}+n_i \tag{2}$$

where $\tilde{x}_i$, $\tilde{y}_i$ and n_i are complex symbols, and n_i represents the complex AWGN whose real and imaginary parts both have zero mean and a variance N_0. Furthermore, θ_i is defined as Wiener model [8], and discrete time Wiener phase noise can be expressed as

$$\theta_{i+1}=\theta_i+w_\theta(i) \tag{3}$$

where $w_\theta(i)$ is stationary Gaussian process with zero mean and a variance σ_θ^2, and independent of n_i. Meanwhile, θ_i and n_i are independent of $\tilde{x}_i$.

Without loss of generality, θ_0 is assumed to be zero and

$$\theta_{i+1}=\sum_{k=0}^{i}w_\theta(k) \tag{4}$$

$E_i=E\left[\|\tilde{x}_i\|^2\right]$ is the average energy per symbol and SNR is $\Pi_{\mathrm{SNR}i}=E_i/(2N_0)$. If θ_i is far less than 1, $\mathrm{e}^{\mathrm{j}\theta_i}$ can be approximated by one-order Maclaurin polynomial