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Analytical approach to sensitivity analysis of flutter speed in bridges considering variable deck mass

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ABSTRACT

Design techniques based upon sensitivity analysis are not usual in the current design of suspension bridges. However, sensitivity analysis has been proved to be a useful tool in the car and aircraft industries. Evaluation of sensitivity analysis is a mandatory step in the way towards an efficient automated optimum design process which would represent a huge jump in the conception of long span bridges. Some of the authors of this paper were pioneers in establishing a methodology for obtaining the sensitivity analysis of flutter speed in suspension bridges a few years ago. That approach was completely analytical and required the evaluation of many matrices related to the phenomenon. In those works the total mass of the deck was considered as constant and such a circumstance supposed a limitation of the method. In the present paper the complete analytical formulation of the sensitivity analysis problem in bridges considering variable deck mass is presented, as well as its application to the design problem of the Great Belt Bridge. Analytical evaluation of sensitivities is a time demanding task, and in order to avoid excessive computation times, distributed computing strategies have been implemented which can be considered as an additional benefit of this approach. For the application example, it has been found that deck cross-section area and torsional inertia are the structural properties with the greatest influence on the flutter performance.

1. Introduction

Nowadays wind engineering for bridges can be considered as a mature discipline. A number of cutting-edge bridges have been built in recent years such as the Akashi Strait Bridge, the Great Belt Bridge, the Stonecutters Bridge, the Sutong Bridge and so on. In these kinds of projects a comprehensive set of wind related responses are taken into consideration, for instance static deformation under wind load, vortex-induced excitation, buffeting and, of course, flutter. In recent times, the focus in the design of cable supported bridges has gradually moved towards service, environmental and aesthetic issues, as well as financial aspects. An example of this trend is the Forth Replacement Crossing project that is being currently developed under the auspices of Transport Scotland.

In some sectors such as the automobile and aeronautic industries where cost-cutting, but also safe performance, are key aspects design based upon sensitivity analysis has become a powerful tool. Sensitivity analysis was first developed in the 1960s and it keeps on being an active area of research. In fact, in a recent paper published in 2008 by Mukherjee et al. [1], it is stated that, "*In the design and analysis of multibody dynamic systems, sensitivity analysis is a critical tool for good design decisions*". Comprehensive references on the subject can be found in [2–4].

The flutter response of a bridge can be determined from complete aeroelastic models tested in boundary layer wind tunnels [5], from sectional model tests employing a set of up to 18 flutter functions obtained by means of wind tunnel tests [6] or alternatively from using indicial functions in time domain [7,8]. Nowadays flutter analysis is commonly carried out in bridge engineering applications in the frequency domain employing a set of empirical functions called flutter derivatives or flutter functions. A number of authors have paid attention to the role played by the flutter functions in the flutter response of slender structures. Chen and Kareem [9] and Chen [10] proposed a closed-form solution to the bimodal flutter problem considering only the flutter functions H_3^*, A_1^*, A_2^* and A_3^*. Also Bartoli and Mannini [11] proposed a simplified approach for the evaluation of classical and torsional flutter which was funded on the key role played by the flutter functions H_1^*, A_2^* and A_3^*. Besides this, the effect of dumping and the frequency ratio were also studied. The branch switch characteristics in coupled flutter and the relative importance of the flutter functions have been studied by Matsumoto and co-workers [12]. In this study the influence of the flutter functions H_3^* and A_1^* in coupled flutter stabilization was highlighted. It is evident that the effect of each individual flutter function on the overall flutter

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performance of slender structures has been extensively analyzed in recent years.

In the present work the authors have adopted a different approach, as they have focused on the contribution of the structural properties of bridge decks to the flutter response. As a matter of fact the set of empirical flutter functions has been considered as invariant, as the cross-sectional geometry is not allowed to undergo modification, on the understanding that the deck aerodynamics has been previously studied. Regarding flutter analysis, the multimode approach solving a complex eigenvalue problem has been chosen for the sake of accuracy and general applicability of the flutter sensitivity analysis formulation. As a consequence, closed-form solutions or simplified formulae have not been taken into consideration. On the other hand time domain formulations, as those based on step by step integration considering indicial functions, were not considered as distributing programming cannot be implemented due to the time-advancing nature of the solution process [13].

Regarding the application of sensitivity analysis in the flutter problem, pioneering investigations carried out by Rudisil and Bhatia [14] and Haftka and Yates [15] were related to aircraft structures and a more recent application can be found in Balis Crema et al. [16]. On the other hand, sensibility analysis applications in bridge flutter can be found in previous publications by the authors [17,18] who applied sensitivity analysis to cable supported bridges assuming constant deck mass. Also Zhang [19,20] has reported parametric analyses on the flutter stability of cable supported bridges. In this paper the formulation of sensitivity analysis of the flutter phenomenon considering variable deck mass is presented, as well as the results obtained from a real model of the Great Belt Bridge.

The authors envision that design based upon sensitivity analysis gives "hands on" information to the designer, allowing him to know in advance and accurately how any change in a structural characteristic affects the performance of the whole structure. For example, a positive sensitivity of a structural response with respect to a design variable means that an increment in the design variable will cause an increase in the response, and that the response increment will be larger as the numerical value of the sensitivity increases.

From a practical point of view, the sensitivity of the flutter speed can be understood as accurate information related in the way a design variable should be modified in order to change the bridge's critical flutter speed with the aim of fulfilling the design flutter requirements. However, assuming that the bridge deck enjoys an efficient aerodynamic design, sensitivity analysis provides designers with precise information regarding the amount of material which can be eliminated from the deck cross-section while keeping flutter speed above the one stipulated in the project requirements. This approach would produce a safe structure (flutter speed higher than the one required in the project) built with an amount of material closer to the optimum, thus a more economical structure.

2. Formulation

Analytical formulation for the sensitivity analysis of the flutter wind speed considering a general design variable (a structural property to be modified) x is going to be presented. It must be borne in mind that this is a comprehensive formulation that can be applied to general long span bridge design. There are alternative approaches such as the popular finite differences. Finite differences have been used by the authors in this research in order to validate the results obtained from applying the analytical formulation for sensitivity analysis reported in this paper. Nevertheless, finite differences present important shortcomings: very long computation time is required, there is an important uncertainty about the adequate step size and extensive checking of the obtained results is mandatory as changes in the order of the mode shapes occur when the design variables are perturbed.

The first step of the methodology to be presented is to develop a three dimensional finite element model of the bridge using beam elements. The model must be sufficiently detailed while limiting the number of degrees of freedom in order to avoid excessive problem sizes.

Suspension bridges manifest an increase in their stiffness with respect to linear theory formulation due to the tensile force in cables. In order to capture this effect, the non-linear structural formulation considering the geometrical stiffness for beam elements formulation has been considered. The main formulae are summarized in Appendix A.

2.1. Sensitivity analysis of natural frequencies and mode shapes

Determination of flutter speed considering the multimode approach requires prior evaluation of the structure's natural frequencies and mode shapes. Therefore, sensitivity analyses of natural frequencies and mode shapes must be obtained as a first step before computing the sensitivity of the flutter velocity.

In order to obtain the structure's natural frequencies and mode shapes the following eigenvector problem must be solved:

$$[\mathbf{K}_{\text{nolin}} - \omega_n^2 \cdot \mathbf{M}] \cdot \boldsymbol{\phi}_n = \mathbf{0} \tag{1}$$

where ω_n is the nth natural frequency, $\boldsymbol{\phi}_n$ is nth mode shape and $\mathbf{M}$ is structure mass matrix.

Deriving Eq. (1) [21] the sensitivity of natural frequencies with respect to a generic design variable x can be written as:

$$\frac{\partial(\omega_n^2)}{\partial x} = \frac{\boldsymbol{\phi}_n^T \cdot \left(\frac{\partial \mathbf{K}_{\text{nolin}}}{\partial x} - \omega_n^2 \cdot \frac{\partial \mathbf{M}}{\partial x}\right) \cdot \boldsymbol{\phi}_n}{\boldsymbol{\phi}_n^T \cdot \mathbf{M} \cdot \boldsymbol{\phi}_n} \tag{2}$$

The sensitivities of the natural mode shapes must be obtained by solving the following system of equations:

$$(\mathbf{K}_{\text{nolin}} - \omega_n^2 \cdot \mathbf{M}) \cdot \frac{\partial \boldsymbol{\phi}_n}{\partial x} = \frac{\partial(\omega_n^2)}{\partial x} \cdot \mathbf{M} \cdot \boldsymbol{\phi}_n - \left(\frac{\partial \mathbf{K}_{\text{nolin}}}{\partial x} - \omega_n^2 \cdot \frac{\partial \mathbf{M}}{\partial x}\right) \cdot \boldsymbol{\phi}_n \tag{3}$$

Special care must be taken when solving the system of Eq. (3) due to its singularity [22].

In both, Eqs. (2) and (3) the derivatives of the non-linear stiffness and the mass matrixes have to be obtained. Calculation of the mass matrix derivative can be carried out using the following expression:

$$\frac{\partial \mathbf{M}}{\partial x} = \mathbf{IE}\left(\frac{\partial \mathbf{M}^{\text{b}}}{\partial x}\right) \tag{4}$$

where $\mathbf{M}^{\text{b}}$ is the element mass matrix.

Analytical evaluation of the derivative of the stiffness matrix in non-linear structural theory with respect to a generic design variable requires a more sophisticated approach. Initially, that derivative can be written as the addition of two terms:

$$\frac{\partial \mathbf{K}_{\text{nolin}}}{\partial x} = \frac{\partial \mathbf{K}_{\text{lin}}}{\partial x} + \frac{\partial \mathbf{K}_{\text{G}}}{\partial x} \tag{5}$$

The derivative of the linear stiffness matrix is:

$$\frac{\partial \mathbf{K}_{\text{lin}}}{\partial x} = \mathbf{IE}\left(\frac{\partial \mathbf{K}_{\text{E}}^{\text{b}}}{\partial x}\right) \tag{6}$$

where $\mathbf{K}_{\text{E}}^{\text{b}}$ is the elementary stiffness matrix in linear theory.