Journal of Sound and Vibration 330 (2011) 4340–4351

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Vibro-acoustic design sensitivity analysis using the wave-based method

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ARTICLE INFO

Article history:
Received 20 December 2010
Received in revised form
7 March 2011
Accepted 31 March 2011
Handling Editor: L.G. Tham
Available online 7 May 2011

ABSTRACT

Conventional element-based methods, such as the finite element method (FEM) and boundary element method (BEM), require mesh refinements at higher frequencies in order to converge. Therefore, their applications are limited to low frequencies. Compared to element-based methods, the wave-based method (WBM) adopts exact solutions of the governing differential equation instead of simple polynomials to describe the dynamic response variables within the subdomains. As such, the WBM does not require a finer division of subdomains as the frequency increases in order to exhibit high computational efficiency. In this paper, the design sensitivity formulation of a semi-coupled structural–acoustic problem is implemented using the WBM. Here, the results of structural harmonic analyses are imported as the boundary conditions for the acoustic domain, which consists of multiple wave-based subdomains. The cross-sectional area of each beam element is considered as a sizing design variable. Then, the adjoint variable method (AVM) is used to efficiently compute the sensitivity. The adjoint variable is obtained from structural reanalysis using an adjoint load composed of the system matrix and an evaluation of the wave functions of each boundary. The proposed sensitivity formulation is subsequently applied to a two-dimensional (2D) vehicle model. Finally, the sensitivity results are compared to the finite difference sensitivity results, which show good agreement.

1. Introduction

The most commonly used numerical prediction techniques for structural–acoustic problems are deterministic element-based methods such as the finite element method (FEM) [1,2] and the boundary element method (BEM) [3,4]. These methods use locally supported simple polynomials as shape functions within the elements. Since these methods have no restrictions regarding the geometric complexity of the considered problem, they are powerful for analyzing a generally shaped model. For example, in the FEM the entire domain must be divided into small elements; the system matrices are composed of real, banded, symmetric, and frequency independent coefficients, such that efficient storage and solution schemes are possible. However, unbounded problems can only be tackled after the introduction of an additional artificial boundary condition, thereby making the FEM not inherently suited for the prediction of unbounded acoustic problems. In contrast, the BEM requires only the boundary discretization of the considered problem, such that the number of element of boundary element (BE) models is smaller than the corresponding finite element (FE) models. It can also handle unbounded

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2011.03.030

problems, because the Sommerfeld radiation condition is inherently satisfied due to the Green functions applied. However, BE matrix coefficients are complex, fully populated, asymmetric, and frequency dependent, thereby making the model construction and solution processes more complicated than for the FEM. Furthermore, the drawback of element-based methods is that the number of elements and subsequent computational efforts increase as the wavelengths shorten at higher frequencies in order to maintain a reasonable prediction accuracy. As such, the prediction results at higher frequencies are substantially affected by interpolation and pollution errors [5]; thus, the practical applicability of element-based methods is limited to low frequencies.

The statistical energy analysis (SEA) [6] is the most widely used method for predicting system response at higher frequencies, using the power flow between subsystems. It is used to estimate the averaged energy response functions of individual subsystems. This statistical approach is computationally efficient, and can be used to predict the dynamic response under uncertainty pertaining to manufacturing tolerances, fabrication imperfections, and the dynamic properties of joints between components. However SEA is not valid for mid-low frequency analyses due to its insufficient modal density for obtaining reliable results [7]. To make up for this weakness, a hybrid method combining the finite element analysis (FEA) and SEA were developed [8,9]. However, compatibility at the joint between the SEA and FEA variables must be considered in order to maintain accuracy. Furthermore, major drawbacks for SEA are that the spatial variation of dynamic responses within a subsystem cannot be represented, and that design variables do not explicitly appear in the governing equation. Therefore, SEA analyses are not generally preferred for use in a design sensitivity analysis (DSA); Ref. [10] gives an overview of other, non-SEA based mid-frequency prediction techniques. One of them is the wave-based method (WBM).

The WBM is a deterministic prediction technique based on an indirect Trefftz approach [11,12]. Dynamic response variables are described by a set of wave functions, which are the exact solutions of the governing differential equations. In the WBM, the main processes of model construction and solution are to determine the contribution factors to make wave functions satisfy the boundary and continuity conditions. Compared to conventional element-based methods, no fine discretization of the domain is necessary, only multiple convex subdomains are required for its convergence—which are independent of the frequencies. As a result, the WBM yields a small number of subdomains and exhibits a high computational efficiency. The efficiency of the WBM at higher frequencies has been shown for various applications in several publications like in Refs. [10,13]. In those publications, the WBM has been shown to be more efficient as compared to commonly used element-based methods. Moreover, every field variable can be represented via a combination of corresponding wave functions and contribution factors, thereby leading to a regular sensitivity equation form.

In this paper, the DSA of a structural–acoustic semi-coupled problem using the WBM is presented. The DSA is the essential part in the gradient-based optimization. There are many published works conducted for DSA and optimization in the structural–acoustic problem. Ma and Hagiwara [14,15] and Choi et al. [16] performed DSA using FEM on the structural–acoustic coupled problem. However, it requires excessive number of 3-D elements predicting the acoustic field variable using FEM. To avoid the discretization of air domain, BEM replaces FEM in the acoustic analysis. Salagame et al. [17] derived analytical sensitivity of acoustic power radiated from plates using analytic method. Wang and Lee [18] developed global acoustic DSA by employing continuum sensitivity. The chain rule regarding the continuum structural sensitivity with acoustic sensitivity was used to obtain accurate and efficient sensitivity results. Lee et al. [19] introduced the topology optimization on boundary element model using genetic algorithm. Kim et al. [20] presented the design sensitivity analysis of structural–acoustic problems using sequential adjoint variable method (AVM). The adjoint variable is calculated from the structural adjoint reanalysis.

Note that the earlier works are the element-based DSA of a structural–acoustic system, whereas there are few papers pertaining to DSA using the WBM. Recently, a DSA for a mid-frequency analysis of a coupled vibro-acoustic problem was developed [21], though it adopts a direct differentiation method (DDM) to obtain the sensitivity. DDM directly solves for the design dependency of a state variable, and then computes the sensitivity using a chain rule. However, the AVM constructs an adjoint problem that solves for the adjoint variable, which contains all implicitly dependent terms. Hence, the AVM is preferable when the number of design variables is larger than active constraints [22]. To this end, most engineering problems are focused on optimizing a particular objective function of a model consisting of many design variables; therefore, the AVM is considered much more efficient than the DDM.

In this paper, the structural dynamic response is obtained using the FEM, and the results are imported as boundary conditions for the wave-based acoustic model. The cross-sectional area of each structural element is considered as a sizing design variable. The adjoint variable is then obtained from a structural reanalysis using an adjoint load comprised of the wave-based system matrix and an evaluation of the wave functions of each boundary location. The sensitivity formulation proposed in this paper is subsequently applied to a generally shaped two-dimensional (2D) vehicle model, and finally the results are compared to finite difference sensitivity results, with good agreement obtained.

2. Structural finite element method

The equation of the dynamic structure under harmonic excitation can be written as [16]

$$-\omega^2 \rho \mathbf{z}(\mathbf{x}) + \mathrm{j}\omega C \mathbf{z}(\mathbf{x}) + A\mathbf{z}(\mathbf{x}) = \mathbf{f}(\mathbf{x}), \quad \mathbf{x} \in \Omega^S, \tag{1}$$

where Ω^S is the structural domain, ω is the angular frequency, ρ and C are the structural mass density and viscous damping effects, respectively, A is a linear partial differential operator that represents the stiffness of the structure, $\mathbf{z}$