Comput. Methods Appl. Mech. Engrg. 241–244 (2012) 142–154

Contents lists available at SciVerse ScienceDirect

Comput. Methods Appl. Mech. Engrg.

journal homepage: www.elsevier.com/locate/cma

Sensitivity analysis with the modified Heaviside function for the optimal layout design of multi-component systems

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ARTICLE INFO

Article history:
Received 18 December 2011
Received in revised form 2 June 2012
Accepted 4 June 2012
Available online 16 June 2012

Keywords:
Multi-component system
Modified Heaviside function
Sensitivity analysis
Topology optimization
Geometric perturbation model
Material perturbation model

ABSTRACT

Two kinds of design variables, i.e., pseudo-density variables associated with the framework structure and location design variables associated with connected components are involved in the layout design of multi-component systems. Although sensitivities with respect to the first ones can easily be carried out as in topology optimization, the semi-analytical method (SAM) is often used for sensitivity analysis with respect to the location design variables. Due to the geometric perturbation of the finite element mesh, the latter can then be regarded as a geometric perturbation model (GPM). In this paper, we propose a material perturbation model (MPM) using fixed finite element (FE) mesh for sensitivity analysis with respect to location design variables. The material discontinuity across the boundary between each component and the framework structure is smoothed approximately by means of a modified Heaviside function. When a location design variable of a certain component is perturbed, attached finite elements to the component boundary are assumed to undertake only a shift of material properties while the finite element mesh itself remains geometrically unchanged. As a result, analytical sensitivities with respect to location design variables are achieved as easily as for pseudo-density variables. The computing efficiency is thus improved because the velocity field for the mesh perturbation in the semi-analytical scheme is no longer needed. The MPM is illustrated by means of numerical tests, especially the design optimization of 3D multi-component systems.

1. Introduction

Topology optimization is one of the most motivated and challenging topics in the past decades and has been successfully applied to a variety of problems including designs for structural compliance, natural frequencies and buckling loads etc. [1]. Different schemes such as the homogenization-based model, solid isotropic material with penalization (SIMP), rational approximation of material properties (RAMP) and level-set method etc. were established [2–7].

The layout design of complex multi-component systems studied here can be regarded as an extension of existing topology optimization and is attracting much attention [8–13]. As shown in Fig. 1(a) and (b), the involved components might act as some functional or electronic devices with specific mechanical properties. When they are considered as non-designable elastic solids or voids with fixed positions, the problem can be dealt with directly by means of the classic topology optimization methodology. However, the challenging design is to perform the location optimization of the involved components and topology optimization of the framework structure simultaneously, as illustrated in Fig. 1(c). On the one hand, the given components should be properly embedded in a limited design space without overlap to satisfy the compactness or other geometric and physical conditions. On the other hand, an optimal configuration of the framework structure has to be figured out inside the system.

If the layout design is concerned with the system compliance minimization, it is necessary to calculate its sensitivities with respect to pseudo-density design variables of the framework structure and location design variables of the components. In previous implementations [10–13], the sensitivities related to pseudo-density variables on the so-called density points are calculated in an analytical way. However, the sensitivities related to location design variables resorted to the semi-analytical approach, which is relatively time-consuming especially for large-scale 3D problems because the mesh shape perturbation has to be determined at each perturbation of each location design variable. In some cases, GPM may also result in mesh distortions and lead to analysis failures. Recently, the superelement technique is adopted to model the components and a sensitivity analysis approach based on it is proposed in favor of computing efficiency [14]. Even though it has been shown that the superelement technique based sensitivity approach greatly improves computing efficiency and is capable of dealing with a large number of components in 2D situation, it is

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http://dx.doi.org/10.1016/j.cma.2012.06.005

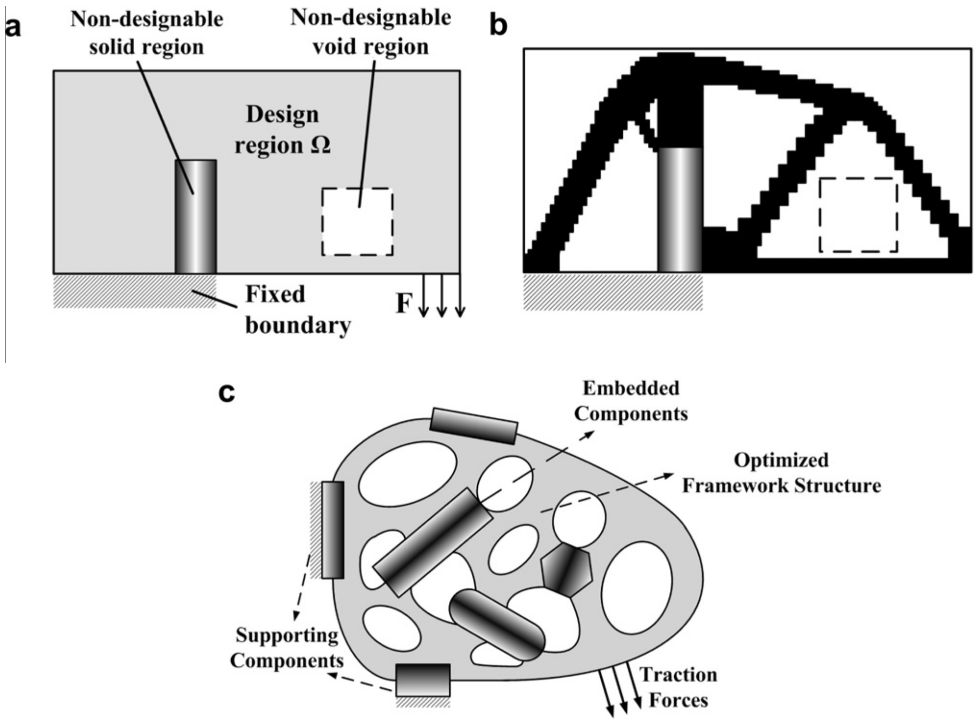

Fig. 1. Illustration of multi-component problem: (a) problem description, (b) topology design optimization and (c) topology optimization with movable components.

still incapable of solving large-scale 3D problems for both reasons of low computing efficiency and parametric programming complexity.

In the structural optimization community, the Heaviside function and its modified versions were widely used in the level-set method [15–16] and density filters of SIMP based methods [17–19]. In level-set method based topology optimizations, the Heaviside function was used to avoid regenerating the element mesh when the boundary changes during the iterative design process. In order to obtain a black and white solution, i.e., pseudo-density design variables converge to 0 or 1, Guest et al. [17] proposed a Heaviside function based density filter strategy. Later, Sigmund reformulated the Heaviside operator and proved that the Heaviside filters provide the best and most discrete designs among all morphology based schemes [18]. Based on the previous work, a volume preserving density filter based on Heaviside functions was proposed for a better efficiency and stability in optimization by Xu et al. [19]. Recently, the Heaviside function was also employed to describe the nonlinear relationship between stress and material modulus to smooth the constitutive discontinuity [20].

In this paper, we present an explicit and analytical sensitivity analysis approach with respect to location design variables based on the modified Heaviside function. The latter is used to smooth approximately the discontinuity of material properties over adjacent elements attaching the boundary of each movable component. Meanwhile, the perturbation effect of each location design variable is modeled as a material shift over a fixed FE mesh, i.e., the so-called MPM. Numerical tests prove that the MPM is easy to implement, reliable and able to improve the optimization efficiency greatly.

2. Mathematical formulation of the integrated layout design optimization

It is recognized that the optimal layout design of multi-component systems is more complicated than the traditional topology optimization problems with a fixed FE mesh over the design domain. The reason lies in the mesh variation iteratively along with the location variation of components. Therefore, the design model has to be managed for its consistence. In our formulation, the general optimization model consists of two sub-optimization models that are layout optimization and topology optimization. Two analysis models, i.e., geometrical model and finite element model are performed to provide geometrical and mechanical responses, respectively. Each of these models serves for its proper role but integrated as a whole.

Consider the design domain with two components, as illustrated in Fig. 2(a). Due to the location variation of components within the limited space, pseudo-density variables used for topology optimization of the framework structure are attributed to fixed density points that are distributed a priori as the centers of a virtual background mesh over the design domain [10]. This is different form the common definition of pseudo-density variables assigned to elements shown in Fig. 2(b). During the design optimization, although the finite element mesh is updated, the density points are fixed. In this way, each element outside the components receives the pseudo-density value from the nearest density point and the elements inside the components receive material property of the components.

Mathematically, suppose η_i is the pseudo-density variable attached to the i th density point that locally dominates q elements. Assume that the elastic modulus for the framework structure and the ε th component is E_f and E_ε, respectively. Based on SIMP scheme, the elastic modulus of the e th element can thus be penalized as:

$$E_{ie} = \begin{cases} \eta_i^p E_f & \Omega_{ie} \not\subset \bigcup_{\varepsilon=1}^{n_c} \Omega_\varepsilon \\ E_\varepsilon & \Omega_{ie} \subset \Omega_\varepsilon \end{cases} \quad (1)$$

where p is the penalty factor set to be 3 as typically used in SIMP based topology optimization [1].