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KALMAN FILTERING CORRECTION IN REAL-TIME FORECASTING WITH HYDRODYNAMIC MODEL*

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Abstract: Accurate and reliable flood forecast is crucial for efficient real-time river management, including flood control, flood warning, reservoir operation and river regulation. In order to improve the estimate of the initial state of the forecasting system and to reduce the errors in the forecast period a data assimilation procedure was often need. The Kalman filter was proven to be an efficient method to adjust real-time flood series and improve the initial conditions before the forecast. A new model integrating the hydraulic model with the Kalman filter for real-time correction of flood forecast was developed and applied in the Three Gorges interzone of the Yangtze River. The method was calibrated and verified against the observed flood stage and discharge during Three Gorges Dam construction periods (2004). The results demonstrate that the new model incorporates an accurate and fast updating technique can improve the reliability of the flood forecast.

Key words: real-time flood forecast, Kalman filter, hydraulic model, alternate Kalman filtering method

1. Introduction

A lot of techniques have been used in real-time updating of deterministic models based on hydraulic routing [1,2]. Compared with other adjustments, the Kalman filtering is an efficient method to adjust real-time flood series because it is based on the unbiased minimum variance estimate so that it can achieve optimal estimation of state variables in the system, meanwhile, dynamically consistent interpolator that is able to update the entire state of a modeling system based on information from measurements [3-6]. Various techniques based on the Kalman filter have been presented in the correction area for short-term forecasting [7].

There are two categories for real-time correction based on hydraulic model by using the Kalman filtering technique. One is to update errors in forecasting period [6]. The other is to couple the Kalman filtering technique with hydraulic model and then update the state variables [8, 9].

This study belongs to the second category. For the real-time flood forecasting with one dimensional hydraulic flow model, there are two different state variables: water stage and discharge. The correction of the forecasting is needed to be made separately, but these two state variables are interdependent, which requires the simultaneous corrections. The alternate Kalman filtering method presented in this article includes two full Kalman filters with water stage and discharge as the state variables separately, which are taken in calculation alternately without sequence and the two full Kalman filters for correction are set up based on the Saint-Venant equations.

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2. Methodology

2.1 *Hydraulic model for flood routing*

The continuity and momentum equations for 1-D gradually varying flow are given as [10,11]:

$$B\frac{\partial Z}{\partial t}+\frac{\partial Q}{\partial x}=q \tag{1a}$$

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}(\frac{\alpha Q^2}{A})+gA\frac{\partial Z}{\partial x}+gA\frac{|Q|Q}{K^2}=qV_X \tag{1b}$$

where q is the lateral inflow per unit channel length, Q is the discharge, A is the cross-sectional area, B is the channel width, Z is water stage, V_X is the longitudinal velocity component of the lateral inflow, which is near zero, K is the hydro-modulus, α is momentum correction coefficient, and g is the gravitatiuonal acceleration.

For numerical modeling, the four-point implicit finite-difference scheme [12] is used to discretize Eq.(1) between the jth and the $(j+1)$th cross sections as follows:

$$Q_{j+1}^{k+1}-Q_j^{k+1}+C_jZ_{j+1}^{k+1}+C_jZ_j^{k+1}=D_j+C_j\bullet$$

$$(Z_{j+1}^k+Z_j^k) \tag{2a}$$

$$E_jQ_j^{k+1}+G_jQ_{j+1}^{k+1}+F_jZ_{j+1}^{k+1}-F_jZ_j^{k+1}=$$

$$H_j(Q_{j+1}^k+Q_j^k) \tag{2b}$$

where j ranges from 1 to L, and k is the number of computation time step. The coefficients of C_j, D_j, E_j, F_j, G_j and H_j can be calculated from the initial data of river water stage and discharge:

$$C_j=\frac{B_{j+1/2}^k\Delta x_j}{2\Delta t},\ D_j=q_{j+1/2}\Delta x_j,$$

$$E_j=\frac{\Delta x_j}{2\Delta t}-(\alpha u)_j^k+\frac{g}{2}(\frac{|u|n^2}{R^{4/3}})_j^k\Delta x_j,$$

$$F_j=(gA)_{j+1/2}^k,\ H_j=\frac{\Delta x_j}{2\Delta t},$$

$$G_j=\frac{\Delta x_j}{2\Delta t}+(\alpha u)_{j+1}^k+\frac{g}{2}(\frac{|u|n^2}{R^{4/3}})_{j+1}^k\Delta x_j$$

where Δx_j is the computational segment along the river, Δt the time interval, u is the fluid velocity, n the friction parameter, and R the hydraulic radius. For all computational sections from 1 to L, Eq.(2) can be integrated as the following linear equations with known coefficients.

$$\boldsymbol{A}\boldsymbol{Q}^{k+1}+\boldsymbol{B}\boldsymbol{Z}^{k+1}=\boldsymbol{C}\boldsymbol{Z}^k+\boldsymbol{U}_1 \tag{3a}$$

$$\boldsymbol{D}\boldsymbol{Q}^{k+1}+\boldsymbol{E}\boldsymbol{Z}^{k+1}=\boldsymbol{F}\boldsymbol{Q}^k+\boldsymbol{U}_2 \tag{3b}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$, $\boldsymbol{E}$ and $\boldsymbol{U}_1$, $\boldsymbol{U}_2$, $\boldsymbol{F}$ are coefficient matrices constituted by coefficients of C_j, D_j, E_j, F_j, G_j and H_j. The unknown water stage $\boldsymbol{Z}^{k+1}$ and discharge $\boldsymbol{Q}^{k+1}$ at time step (k+1) are expressed with known variables at time step k, resulting in

$$\boldsymbol{Z}^{k+1}=(-\boldsymbol{D}\boldsymbol{A}^{-1}\boldsymbol{B}+\boldsymbol{E})^{-1}(-\boldsymbol{D}\boldsymbol{A}^{-1}\boldsymbol{C})\boldsymbol{Z}^k+$$

$$(-\boldsymbol{D}\boldsymbol{A}^{-1}\boldsymbol{B}+\boldsymbol{E})^{-1}(\boldsymbol{F}\boldsymbol{Q}^k-\boldsymbol{D}\boldsymbol{A}^{-1}\boldsymbol{U}_1+\boldsymbol{U}_2) \tag{4a}$$

$$\boldsymbol{Q}^{k+1}=(\boldsymbol{A}-\boldsymbol{B}\boldsymbol{E}^{-1}\boldsymbol{D})^{-1}(-\boldsymbol{B}\boldsymbol{E}^{-1}\boldsymbol{F})\boldsymbol{Q}^k+$$

$$(\boldsymbol{A}-\boldsymbol{B}\boldsymbol{E}^{-1}\boldsymbol{D})^{-1}(\boldsymbol{C}\boldsymbol{Z}^k-\boldsymbol{B}\boldsymbol{E}^{-1}\boldsymbol{U}_2+\boldsymbol{U}_1) \tag{4b}$$

which are simplified as

$$\boldsymbol{Z}^{k+1}=\boldsymbol{\Phi}_Z^k\boldsymbol{Z}^k+\boldsymbol{U}_Z^k \tag{5}$$

$$\boldsymbol{Q}^{k+1}=\boldsymbol{\Phi}_Q^k\boldsymbol{Q}^k+\boldsymbol{U}_Q^k \tag{6}$$

These equations can be used for flood forecasting on the basis of the calculated coefficient matrices $\boldsymbol{\Phi}_Z^k$, $\boldsymbol{U}_Z^k$, $\boldsymbol{\Phi}_Q^k$ and $\boldsymbol{U}_Q^k$ when the friction parameters, initial and boundary conditions of water stage and discharge are known.

2.2 *Kalman filter*

The Kalman filter can be used in any kind of linear stochastic system for adjusting both stochastic measurement error and dynamic model error [13]. The Kalman filter can be described by a system state equation and a measurement equation [14,15] given by

System state equation:

$$\boldsymbol{X}_{k+1}=\boldsymbol{\Phi}_{k+1|k}\boldsymbol{X}_k+\boldsymbol{\Psi}_{k+1|k}\boldsymbol{U}_k+\boldsymbol{\Gamma}_{k+1|k}\boldsymbol{w}_k \tag{7}$$

Measurement equation: