Physica A 392 (2013) 295–306

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Stochastic sensitivity analysis of noise-induced intermittency and transition to chaos in one-dimensional discrete-time systems

Irina Bashkirtseva, Lev Ryashko *

Ural Federal University, Pr. Lenina 51, Ekaterinburg, Russia

ARTICLE INFO

Article history:
Received 13 April 2012
Received in revised form 20 August 2012
Available online 7 September 2012

Keywords:
Intermittency
Noise-induced chaos
Stochastic sensitivity function
Tangent bifurcation

ABSTRACT

We study a phenomenon of noise-induced intermittency for the stochastically forced one-dimensional discrete-time system near tangent bifurcation. In a subcritical zone, where the deterministic system has a single stable equilibrium, even small noises generate large-amplitude chaotic oscillations and intermittency. We show that this phenomenon can be explained by a high stochastic sensitivity of this equilibrium. For the analysis of this system, we suggest a constructive method based on stochastic sensitivity functions and confidence intervals technique. An explicit formula for the value of the noise intensity threshold corresponding to the onset of noise-induced intermittency is found. On the basis of our approach, a parametrical diagram of different stochastic regimes of intermittency and asymptotics are given.

1. Introduction

Due to the interaction between nonlinearity and stochasticity, noise can induce a number of interesting unexpected phenomena in dynamical systems, such as noise-induced transitions [1,2], noise-induced resonance [3–5], noise-induced excitement [6], noise-induced order [7,8] and chaos [9,10]. The transition to chaos is a fundamental and widely studied problem in deterministic nonlinear dynamics. Among the possible routes to chaos is an intermittency route. The system demonstrating intermittent behavior remains for a long duration in some regular regime (laminar state) and at unpredictable moments begins to exhibit chaotic oscillations (turbulent state) before returning to the laminar state. Pomeau and Manneville [11,12] have proposed a simple deterministic one-dimensional model and classified three different types of intermittency. These types (I, II and III) correspond to a tangent bifurcation, a subcritical Hopf bifurcation, or an inverse period-doubling bifurcation. A renormalization group approach to analyze type-I intermittency has been used in Refs. [13,14].

In this paper, we focus on the study of the noise-induced type-I intermittency phenomenon. An influence of noise on the intermittent behavior of nonlinear dynamical systems has been widely studied [15–21].

Frequently, noise-induced intermittency is caused by the multistability of the initial nonlinear deterministic system. Indeed, let the system have coexisting regular (equilibrium or limit cycle) and chaotic attractors. Due to random disturbances, a phase trajectory can cross a separatrix between basins of the attraction and exhibit a new dynamical regime which has no analog in the deterministic case. Random trajectories hopping between basins of coexisting deterministic attractors form a new stochastic attractor. This stochastic attractor joins together two types of dynamics. Trajectories in this attractor exhibit the alternation of phases of noisy regular and noisy chaotic dynamics near initial deterministic attractors and define corresponding type of noise-induced intermittency.

* Corresponding author.
E-mail address: lev.ryashko@usu.ru (L. Ryashko).

0378-4371/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2012.09.001

However, the multistability is not an obligatory condition of the noise-induced intermittency. The phenomenon of noise-induced intermittency can be observed in the specific dynamical systems with a single stable equilibrium only. For these systems, a basin of attraction of equilibrium can be separated on two zones. If the initial point belongs to the first zone localized near the equilibrium, the system quickly relaxes back into the stable equilibrium. Once the initial point lies in second zone, a large excursion of the trajectory is observed. In this case, the system demonstrates high-amplitude oscillations until the trajectory returns to the first zone. Under the small random disturbances, trajectory of this type system leaves a stable equilibrium and forms some probabilistic distribution around it. This noisy equilibrium is localized in the first zone. Once the noise intensity exceeds a certain threshold, the random trajectory hits at second zone and exhibits long-time noisy oscillations until return to first zone and so on. In such a way the stochastically forced system with super-threshold noise demonstrates noise-induced intermittency. Under the random disturbances, this system is transformed from order to chaos. The standard model with this type noise-induced intermittency is a one-dimensional map in a zone of tangent bifurcation. Similar phenomena when small noises generate large-amplitude oscillations can also be observed in continuous-time systems with a single stable equilibrium. The FitzHugh–Nagumo model is a well known example of such noise-induced excitement [22,6].

A probabilistic analysis of the noise-induced phenomena is based on the investigation of corresponding stochastic attractors. A detailed description of stochastic attractors for continuous-time systems is given by the Kolmogorov–Fokker–Planck equation. For discrete-time systems, this description is given by the corresponding integral equation with Frobenius–Perron operator. However, a direct usage of these equations is very difficult even for the simplest cases. To avoid this complexity, various asymptotics and approximations can be considered [23,24].

A stochastic sensitivity function (SSF) method has been used for the constructive probabilistic description of stochastic attractors for both continuous [25] and discrete-time [26] systems. The aim of our work is to demonstrate how the SSF technique can be applied to the parametrical analysis of the noise-induced intermittency for discrete-time nonlinear systems. Our general approach is illustrated on the example of the simple one-dimensional model.

In Section 2, we introduce this model and discuss phenomena of noise-induced intermittency and noise-induced chaotization in a subcritical zone near the tangent bifurcation.

The main results of our paper are shown in Section 3.

In Section 3.1, we present a brief theoretical background of the general SSF technique for stochastic equilibria of discrete-time dynamical systems. A constructive description of the dispersion of random states in the stochastic equilibria is given by confidence intervals. The size of the confidence interval is defined by the noise intensity, value of stochastic sensitivity and fiducial probability.

In Section 3.2, this technique is applied to the detailed parametrical analysis of noise-induced intermittency for the one-dimensional model introduced in Section 2. Through this study, we find an explicit formula for the value of noise intensity threshold corresponding the onset of noise-induced intermittency and construct a parametrical diagram of different stochastic regimes.

In Section 3.3, constructive abilities of our approach for the asymptotic analysis of the noise-induced intermittency in a tangent bifurcation zone for the general one-dimensional systems are demonstrated.

2. Phenomena of noise-induced intermittency and chaotization

2.1. Deterministic model. Intermittency

We consider a discrete-time nonlinear dynamic system

$$x_{t+1} = f(x_t, \mu), \quad f(x, \mu) = \mu x(1-x)(lx^2 + px + q), \tag{1}$$

where

$$l = \frac{1}{1 - s_1 + s_2 - s_3}, \qquad p = l(1 - s_1), \qquad q = l(1 - s_1 + s_2),$$
$$s_1 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3, \qquad s_2 = \bar{x}_1\bar{x}_2 + \bar{x}_2\bar{x}_3 + \bar{x}_3\bar{x}_1, \qquad s_3 = \bar{x}_1\bar{x}_2\bar{x}_3.$$

For any μ, the system (1) has a trivial equilibrium $\bar{x}_0 = 0$. Values $\bar{x}_1, \bar{x}_2, \bar{x}_3 (\bar{x}_1 \le \bar{x}_2 \le \bar{x}_3)$ are nontrivial equilibria of the system (1) for $\mu = 1$. As parameter μ varies near $\mu = 1$, these equilibria change too. So we denote the corresponding functions by $\bar{x}_1(\mu), \bar{x}_2(\mu), \bar{x}_3(\mu)$.

This system is a convenient model for the study of the phenomenon of intermittency.

We fix values $\bar{x}_1 = \bar{x}_2 = 0.25$, $\bar{x}_3 = 0.85$ and vary the parameter μ near the value $\mu_* = 1$. Note that for the interval $\mu_* - 0.1 < \mu < \mu_* + 0.1$, the equilibria $\bar{x}_0$ and $\bar{x}_3(\mu)$ are unstable.

Here, the value $\mu_* = 1$ is a tangent bifurcation point (see Fig. 1). For $\mu < \mu_*$, we have $\bar{x}_1(\mu) < \bar{x}_2(\mu)$, where the equilibrium $\bar{x}_1(\mu)$ is stable (black circle) and equilibrium $\bar{x}_2(\mu)$ is unstable (white circle). For $\mu = \mu_*$, these equilibria coalesce into the single semistable equilibrium $\bar{x}_1(\mu_*) = \bar{x}_2(\mu_*)$. For $\mu > \mu_*$, this equilibrium disappears.

In Fig. 2, the attractors of the system (1) for $\mu \in (0.995, 1.005)$ are presented.