Journal of Macroeconomics 31 (2009) 523–533

Contents lists available at ScienceDirect

Journal of Macroeconomics

journal homepage: www.elsevier.com/locate/jmacro

A reputation strategic model of monetary policy in continuous-time

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ARTICLE INFO

Article history:
Received 25 April 2007
Accepted 18 December 2008
Available online 31 December 2008

JEL classification:
E52
E58

Keywords:
Monetary policy
Time consistency problem
Reputation
Continuous model

ABSTRACT

This paper develops a reputation strategic model of monetary policy with a continuous finite or infinite time horizon. By using the optimal stopping theory and introducing the notions of sequentially weak and strong rational expectation equilibria, we show that the time inconsistency problem may be solved with trigger reputation strategies not only for stochastic but also for non-stochastic settings even with a finite horizon. We show the existence of stationary sequentially strong rational expectation equilibrium under some condition, and there always exists a stationary sequentially weak rational expectation equilibrium. Moreover, we investigate the robustness of the sequentially strong rational expectation equilibrium behavior solution by showing that the imposed assumption is reasonable.

1. Introduction

Time inconsistency is an interesting problem in macroeconomics in general, and monetary policy in particular. Although technologies, preferences, and information are the same at different time, the policymaker's optimal policy chosen at time t_1 differs from the optimal policy for t_1 chosen at t_0. The study of time inconsistency is important. It provides positive theories that help us to understand the incentives faced by policymakers and provide the natural starting point for attempts to explain the actual behavior of policymakers and actual policy outcomes.

This problem was first noted by Kydland and Prescott (1977). Several approaches have been proposed to deal with this problem since then. Barro and Gordon (1983) were the first to build a game model to analyze "reputation" of monetary policy. Backus and Driffill (1985) extended the work of Barro and Gordon to a situation in which the public is uncertain about the preferences of the government. Persson and Tabellini (1990) gave an excellent summarization of these models. Al-Nowaihi and Levine (1994) discussed reputation equilibrium in the Barro–Gordon monetary policy game. Haubrich and Ritter (2000) considered the decision problem of a policymaker or government who chooses repeatedly between rule and discretion in the Barro–Gordon model. The second approach is based on the incentive contracting design to monetary policy. Persson and Tabellini (1993), Walsh (1995), Sevensson (1997), and Tian (2005) developed models using this approach. The third approach is built on the legislative governance. The major academic contribution in this area was by Rogoff (1985). Among these approaches, the "reputation" problem is key. If reputation consideration discourages the monetary authorities from

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0164-0704/$ - see front matter
doi:10.1016/j.jmacro.2008.12.003

attempting surprise inflation, then legal or contracting constraints on monetary authorities are unnecessary and may be harmful.

The main questions on reputation are when and how the government chooses inflation optimally to minimize welfare loss, and, whether the punishment on reputation loss can induce the government to keep zero-inflation. The conclusions of Barro–Gordon discrete-time models are: First, there exists a zero-inflation Nash equilibrium if the punishment for the government deviating from zero-inflation is large enough. However, this equilibrium is not sequentially rational over a finite time horizon. The only sequentially rational expectation equilibrium is achieved if the government chooses discretionary inflation and the public expects it. Only over an infinite time horizon can one get a low-inflation equilibrium. Otherwise, the government would be sure in the last period to produce the discretionary outcome whatever the public's expectation were and, working backward, would be expected to do the same in the first period. Secondly, there are multiple Nash equilibria and there is no mechanism to choose among them.

Finite time horizon games are more reasonable than infinite ones in the real world. At least we see a government's terms are limited. Many experimental studies of games suggest that there are cooperation equilibrium when the players are told that the game will end. Consequently, how to induce cooperative behavior in a finitely repeated game is an interesting problem even for game theorists. Also, discrete-time methods have some limitations for reputation games. Fudenberg and Levine (1992) showed that, in every equilibrium, the government's payoff when he uses a discretionary rule becomes at least as hight as the payoff when he keeps the inflation rate at zero. Cripps et al. (2004) showed that reputation effect is temporary. In addition, in the certainty setting with discrete-time, a reputation equilibrium is possible only if the horizon is infinite.

This paper extends the basic Barro–Gordon model of dynamic inconsistent monetary policy with reputation to continuous-time. Continuous-time models permit a sharp characterization of the thresholds that trigger an adjustment and the level to which adjustment is made. One particularly nice result in the paper is to get reputation to work in finite time. In contrast to Barro–Gordon model, we assume that output shocks follow a Brownian motion. As Stokey (2006) pointed out, the economic effects of an aggregate productivity shock depend on the investment and hiring/firing response of firms. In situations where fixed costs are important, continuous-time models in which the stochastic shocks follow a Brownian motion or some other diffusion have strong theoretical appeal. Also, the fact that many price changes are large in magnitude suggests that the shock should be a diffusion process.

We study the time inconsistency problem in monetary policy with the continuous finite or infinite time horizon model by using the optimal stopping theory in the stochastic differential equations literature.[1] The optimal stopping theory can cover many dynamic economic applications under uncertainty. The optimal stopping theory, though relatively complete in its theoretical development, has not yet been widely applied in economics. By using the optimal stopping theory and introducing the notions of sequentially weak and strong rational expectation equilibria, we show that the time inconsistency problem may be solved with trigger reputation strategies within our setting not only for stochastic but also for non-stochastic settings even with a finite horizon. We provide the conditions for the trigger reputation strategy to be a stationary sequentially strong rational expectation equilibrium, i.e., the government will keep the inflation rate at zero when the public's behavior is characterized by strong rational expectation and uses the trigger strategy. We further show that there always exists a stationary sequentially weak rational expectation equilibrium at which the government will keep the inflation at zero.

The results obtained in the paper are sharply contrasted to the negative results from the certainty setting with a discrete-time horizon. Our results on the existence of the stationary zero-inflation policy as an equilibrium solution are also true for the non-stochastic continuous finite horizon settings, which demonstrate the advantage of our continuous-time model compared to the non-stochastic discrete-time finite horizon model discussed in the literature. Thus, a striking advantage of using a continuous-time formulation is that it yields a solution to the time inconsistency problem whereas a discrete-time counterpart does not. Why does the much more complicated continuous-time formulation yield a positive result that the discrete-time formulation could not? Intuitively speaking, it is because, in continuous-time, the government has an option to change a policy in any instant time while, in the discrete-time formulation, the government can change a policy only in each integer time. The solution to the continuous-time formulations can be viewed as the sum of the solutions to the discrete-time formulations for infinitely many small stopped subintervals. Thus, the embedded option in continuous-time formulation may appear to explain why the continuous-time formulation can yield a solution to the time inconsistence while the discrete-time versions in the existing literature fails.[2] Other work on time consistency in continuous-time can also found in Faingold and Sannikov (2007) and Haubrich and Ritter (2004).

We also investigate the robustness of the equilibrium behavior by showing that the imposed assumption is reasonable. We can always expect a stationary zero-inflation outcome by the sequentially strong rational expectation behavior so that the rational expectation reputation can discourage the monetary authority from attempting surprise.

The remainder of the paper is organized as follows. Section 2 will set up the model and provides a solution for the optimal stopping problem faced by the government. In Section 3, we study the equilibrium behavior. The robustness of this monetary game is discussed in Section 4. Section 5 gives the conclusion. All the proofs and how to solve the optimal stopping problem are given in Appendix A.

[1] The discussion about the optimal stopping theory can be found in Friedman (1979) and Øksendal (1998), Tsitsiklis and Van Roy (1999).

[2] Such an advantage of the continuous-time formulations can be also found in other fields such as the principal-agent literature. For instance, Holmstrom and Milgrom's (1987) continuous-time Brownian model not only generate the second-best solution, but their solution is remarkably simple. Schättler and Sung (1997) provided the above explanation for the principal-agent models.