# Stochastic production planning for a biofuel supply chain under demand and price uncertainties

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## HIGHLIGHTS

► The proposed stochastic model outperforms the deterministic model.
► The price of biofuel is modeled as Geometric Brownian Motion (GBM).
► The proposed model can be applied in any biofuel supply chain.

## ARTICLE INFO

## ABSTRACT

In this paper, we propose a stochastic production planning model for a biofuel supply chain under demand and price uncertainties. The supply chain consists of biomass suppliers, biofuel refinery plants and distribution centers. A stochastic linear programming model is proposed within a single-period planning framework to maximize the expected profit. Decisions such as the amount of raw materials purchased, the amount of raw materials consumed and the amount of products produced are considered. Demands of end products are uncertain with known probability distributions. The prices of end products follow Geometric Brownian Motion (GBM). Benders decomposition (BD) with Monte Carlo simulation technique is applied to solve the proposed model. To demonstrate the effectiveness of the proposed stochastic model and the decomposition algorithm, a representative supply chain for an ethanol plant in North Dakota is considered. To investigate the results of the proposed model, a simulation framework is developed to compare the performances of deterministic model and proposed stochastic model. The results from the simulation indicate the proposed model obtain higher expected profit than the deterministic model under different uncertainty settings. Sensitivity analyses are performed to gain management insight on how profit changes due to the uncertainties affect the model developed.

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## 1. Introduction

Today's energy consumption is increasing tremendously. Recent studies have shown that mainstream crude oil cannot sustain the volume of worldwide demand and consumption of energy [1–4]. Renewable energy, specifically biofuel, has gained attention as a competitor and alternative source of energy to crude oil, especially in the transportation sector. In order to ensure a consistent and a competitive supply of these biofuels to the distribution centers, a reliable and resilient supply chain is needed to help coordinate and streamline the demand and supply activities. Literature that has considered biofuel supply chain has not extensively incorporated uncertainties into the supply chain decision-making [5–10]. Incorporation of uncertainties in the supply chain decision-making process helps to make better decisions in realizing the overall objective of the supply chain [11–13]. However, most of the applications in the biofuel supply chain have focused on deterministic problems, such as network optimization and plant location problems by using mixed integer linear programming (MILP) methods [14–23]. Not enough attention has been given to incorporate demand, production, price and other forms of uncertainties in the supply chain decision-making process. Decisions based on deterministic assumptions will result in non-optimal solutions if uncertainties exist. In the biofuel supply chain system, multiple uncertainties such as demands and prices of end products exist; therefore it is essential to develop an optimization model in the biofuel supply chain decision-making that considers existing uncertainties.

Uncertainties in the supply chain have attracted a lot of attention because of its importance in decision-making, and biofuel supply chain uncertainties are not an exception. These uncertainties can be incorporated at the strategic, tactical and operational decision-making levels within the supply chain. Accurately incorporating uncertainties into the biofuel supply chain will result in better

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**Nomenclature**

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| | |
|---|---|
| $i$ | represents supply sources of raw materials $i = 1, 2, \ldots I$ |
| $j$ | represents the product $j = 1, 2, \ldots J$ |
| $k$ | represents the refinery $k = 1, 2, \ldots K$ |
| $c$ | represents the customer distribution centers $c = 1, 2, \ldots C$ |

*Decision variables*

| | |
|---|---|
| $x_{i,k}$ | amount of biomass raw materials from supply source $i$ to plant $k$ |
| $s_{j,k}$ | amount of product $j$ sold at plant $k$ |
| $z_{j,k}$ | amount of product $j$ produced at plant $k$ |
| $v_{i,k}$ | amount of raw materials from supply source $i$ consumed at plant $k$ |
| $R_{i,j}$ | amount of raw materials inventory from supply source $i$ for product $j$ |
| $F_{j,k}$ | amount of end products $j$ at plant $k$ |
| $L_{j,k}$ | amount of lost sales of product $j$ at plant $k$ |
| $B_{j,k}$ | amount of backlog of product $j$ at plant $k$ |

*Parameter*

| | |
|---|---|
| $P_{j,k}$ | selling price of product $j$ at plant $k$ |
| $\theta_{i,k}$ | available biomass from the supply source $i$ at plant $k$ |
| $\Delta_{j,k}$ | conversion factor for the end product $j$ at plant $k$ |
| $y_{i,k}$ | unit cost of raw materials from supply source $i$ at plant $k$ |
| $\tau_{i,k}$ | unit transportation cost of raw materials from supply source $i$ to plant $k$ |
| $\tau_{k,c}$ | unit transportation cost of end products from plant $k$ to demand point $c$ |
| $d_{i,k}$ | transportation distance of raw materials from supply source $i$ to plant $k$ |
| $d_{k,c}$ | transportation distance of end products from plant $k$ to demand point $c$ |
| $F_{j,k}^0$ | end products inventory for previous period at plant $k$ for product $j$ |

| | |
|---|---|
| $R_{i,k}^0$ | raw material inventory for previous period at plant $k$ for raw material from source $i$ |
| $B_{j,k}^0$ | amount of backlog for previous period at plant $k$ for product $j$ |
| $L_{j,k}^0$ | amount of lost sale for previous period at plant $k$ for product $j$ |
| $\beta_{i,t}$ | cost for processing raw material from supply source $i$ |
| $m_{j,k}$ | cost for lost sales for product $j$ at plant $k$ |
| $q_{j,k}$ | cost for backlog for product $j$ at plant $k$ |
| $D_{j,k}$ | demand for product $j$ at plant $k$ |
| $v_{i,k}$ | amount of raw materials consumed from supply source $i$ at plant $k$ |
| $\gamma_{j,k}$ | amount of fractional lost in demand for product $j$ at plant $k$ |
| $S_0$ | initial spot price of the end products |

*Stochastic variables and terms*

| | |
|---|---|
| $\xi$ | scenario representation for the stochastic variable |
| $p_\xi$ | probability of the scenario of each stochastic variable |
| $S_{j,k,\xi}$ | stochastic sales amount of product $j$ at plant $k$ based for the scenario $\xi$ |
| $B_{j,k,\xi}$ | stochastic backlog for previous period at plant $k$ for product $j$ based on the scenario $\xi$ |
| $F_{j,k,\xi}$ | stochastic end products inventory in previous period at plant $k$ for product $j$ scenario $\xi$ |
| $L_{j,k,\xi}$ | stochastic lost sale for previous period at plant $k$ for product $j$ based on the scenario $\xi$ |
| $P_{j,k,\xi}$ | stochastic price of finished goods of product $j$ at plant $k$ for the scenario $\xi$ |
| $S_t$ | calculated spot price of the end products after the price scenarios have been generated |
| $l$ | iteration steps for benders decomposition |
| $N$ | total number of scenarios generated |

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decision-making and give significant improvement of the expected profit and cost. Although this is crucial within the entire supply chain decision-making process, most models that have discussed biofuel supply chain have not discussed these uncertainties extensively. This paper combines both.

The objective of this paper is to maximize the profit of a multi-product, single-period, three-echelon supply chain system subjected to uncertainties in demands and prices of end products. The problem is modeled as a stochastic programming problem, with key decisions such as products production volume, amount of raw material purchased, and the amount of raw materials consumed. To solve the stochastic problem, the decision variables are separated into first-and second-stage decisions. The first-stage decisions are the initial amount of raw materials purchased, volumes end products to be produced, and the raw material consumed. Decisions such as the amount of end products sold, backlog, and lost sales are considered as second-stage decisions. This means postponing the rest of the decisions for the next period after the realization of the uncertainty. The Benders decomposition with Monte Carlo simulation technique is used to solve the proposed model. To demonstrate the effectiveness of the proposed stochastic models and decomposition algorithm, a realistic representative biofuel supply chain in North Dakota is presented.

The rest of this article is organized as follows: Section 2 gives a summary of the problem statement. Section 3 presents the deterministic model. In Section 4, the proposed stochastic models are

presented. In Section 5, the Benders decomposition with simulation algorithm is discussed. Section 6 provides the numerical experimental design and analyses. Final conclusions and future work are discussed in Section 7.

## 2. Problem statement

This paper studies a biofuel supply chain as shown in Fig. 1. The supply chain consists of three layers: biomass raw materials sources, biofuel refinery plants, and distribution centers. There are *i* number of raw material sources, *k* number of plants, and *c* number of distribution centers. The number of end products of biofuel refinery are represented by the term *j*. Representation set for the probability, and number of scenarios, are expressed as *p*, and *ξ* respectively. Biomass raw materials are transported from the sources of raw materials (via truck or rail) to the biofuel plants. Blending of ethanol and the sales of the products take place at the biofuel plants and demand locations respectively. Demands for these products are imposed by external customers. Depending on the producer's option, the products are either sold directly or traded on the Chicago Merchantile Exchange (CMEX).

Meanwhile, initial inventory at the plants for both raw materials and end products are given. Costs, such as inventory holding, backlog and lost sales are added at the producer's expense. Demands of end products are random with known probability distributions. The prices of end products follow the Geometric Brownian Motion (GBM).