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Regional sensitivity analysis using revised mean and variance ratio functions

Pengfei Wei, Zhenzhou Lu*, Wenbin Ruan, Jingwen Song

Northwestern Polytechnical University, School of Aeronautics, Xi'an, Shaanxi 710072, China

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A B S T R A C T

The variance ratio function, derived from the contribution to sample variance (CSV) plot, is a regional sensitivity index for studying how much the output deviates from the original mean of model output when the distribution range of one input is reduced and to measure the contribution of different distribution ranges of each input to the variance of model output. In this paper, the revised mean and variance ratio functions are developed for quantifying the actual change of the model output mean and variance, respectively, when one reduces the range of one input. The connection between the revised variance ratio function and the original one is derived and discussed. It is shown that compared with the classical variance ratio function, the revised one is more suitable to the evaluation of model output variance due to reduced ranges of model inputs. A Monte Carlo procedure, which needs only a set of samples for implementing it, is developed for efficiently computing the revised mean and variance ratio functions. The revised mean and variance ratio functions are compared with the classical ones by using the Ishigami function. At last, they are applied to a planar 10-bar structure.

1. Introduction

Sensitivity analysis (SA) is a study of how “uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input” [1]. It is a scientific model analysis technique for reducing the uncertainty of model output, lowering the risk of policy decision and other purposes, thus it plays an important role in modern science. Generally, the SA techniques presented in the literature can be divided into three groups: local SA, global SA and regional SA.

The local sensitivity indices are defined as the partial derivatives of model output with respect to the model inputs at given points. They have the advantage of computational efficiency, thus have drawn widely attentions of researchers and analysts in the early study [2,3]. However, the local sensitivity indices only reflect the sensitivity information of model output to the inputs at one given point, but cannot tell the sensitivity of model output uncertainty to the full or specific distribution ranges of the inputs.

The global SA aims at measuring the contribution of one input or a set of inputs to the uncertainty of model output. By decreasing the uncertainty of the most influential input identified by the global sensitivity indices, the uncertainty of model output can be reduced efficiently, and by decreasing the uncertainty of the non-influential inputs, the uncertainty of model output cannot be reduced obviously. During the past few decades, researchers have developed many techniques for this purpose such as the elementary effect (EE) method developed by Morris [4] and then improved by Campolongo et al. [5], the moment-independent one proposed by Borgonovo [6], the variance-based one developed by Sobol [7] and Homma and Saltelli [8], and the derivative-based one proposed by Sobol and Kucherenko [9]. Among all these methods, the variance-based one is the most popular, and has been studied widely in the past few decades [10–16]. The main disadvantage of global SA is that it cannot tell us how much the uncertainty of model output can be reduced when the distribution ranges of model inputs are reduced to specific ones.

Regional SA focuses on quantifying the contribution of the distribution ranges of inputs to the uncertainty of model output, and computing the amount of uncertainty reduction that can be obtained when the distribution ranges of model inputs are reduced. In Ref. [17], Bolado-Lavin et al. introduced a regional SA technique called “contribution to sample mean (CSM) plot”, which aims at measuring the effect of the distribution range of an individual input on the mean of model output. It has extensive applications in many areas, such as improving the reliability of engineering structure. Inspired by this work, Tarantola et al. presented another regional SA technique called “contribution to sample variance (CSV) plot” for quantifying how much the output deviates from the model output mean when one reduced the rage of one input [18]. Based on the CSV plot, Tarantola et al. derived

* Corresponding author. Tel./fax: +86 29 88460480.
E-mail address: zhenzhoulu@nwpu.edu.cn (Z. Lu).

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the variance ratio function. Both the CSV and the variance ratio function are defined with respect to the constant mean of model output computed over the full distribution ranges of all inputs, thus they can tell the analysts how much the output deviates from the original model output mean when the range of one input is reduced. However, it is for this reason that they cannot tell the actual reduction of model output variance due to reduced ranges of model inputs when the model output mean varies significantly with respect to the reduced ranges. In Ref. [19], the authors presented the moment-independent regional SA technique for measuring the contribution of the different distribution ranges of each input to the uncertainty of model output characterized by the shift between conditional and unconditional probability density functions (PDFs).

In this paper, we firstly present the revised mean and variance ratio functions. Compared with the classical variance ratio function, the revised one is defined with respect to the mean over the reduced range but not the full ranges of all inputs, thus it is more suitable for the measurement of the reduction of model output variance due to the reduced ranges of model inputs. By using the 3D plots of the revised mean and variance ratio functions, one can learn the amount of reduction of the mean and variance of model output due to any reduced ranges of model inputs. It is also found that the diagonal lines of the mean and variance ratio functions are in fact the conditional expectation and variance, respectively, thus they can be used for establishing metamodel for the computational model and computing the variance-based sensitivity indices. This observation also reveals the intrinsic links between the global and regional SA. Then, a Monte Carlo procedure is introduced for efficiently computing the revised mean and variance ratio functions. Only one set of samples is needed for implementing the algorithm, thus no extra computational burden is introduced.

The rest of this paper is organized as follows. Section 2 reviews the CSM and CSV plots and the classical variance ratio function. The revised mean and variance ratio functions are presented and interpreted in Section 3. The classical and revised variance ratio functions are compared using the Ishigami function in Section 4.

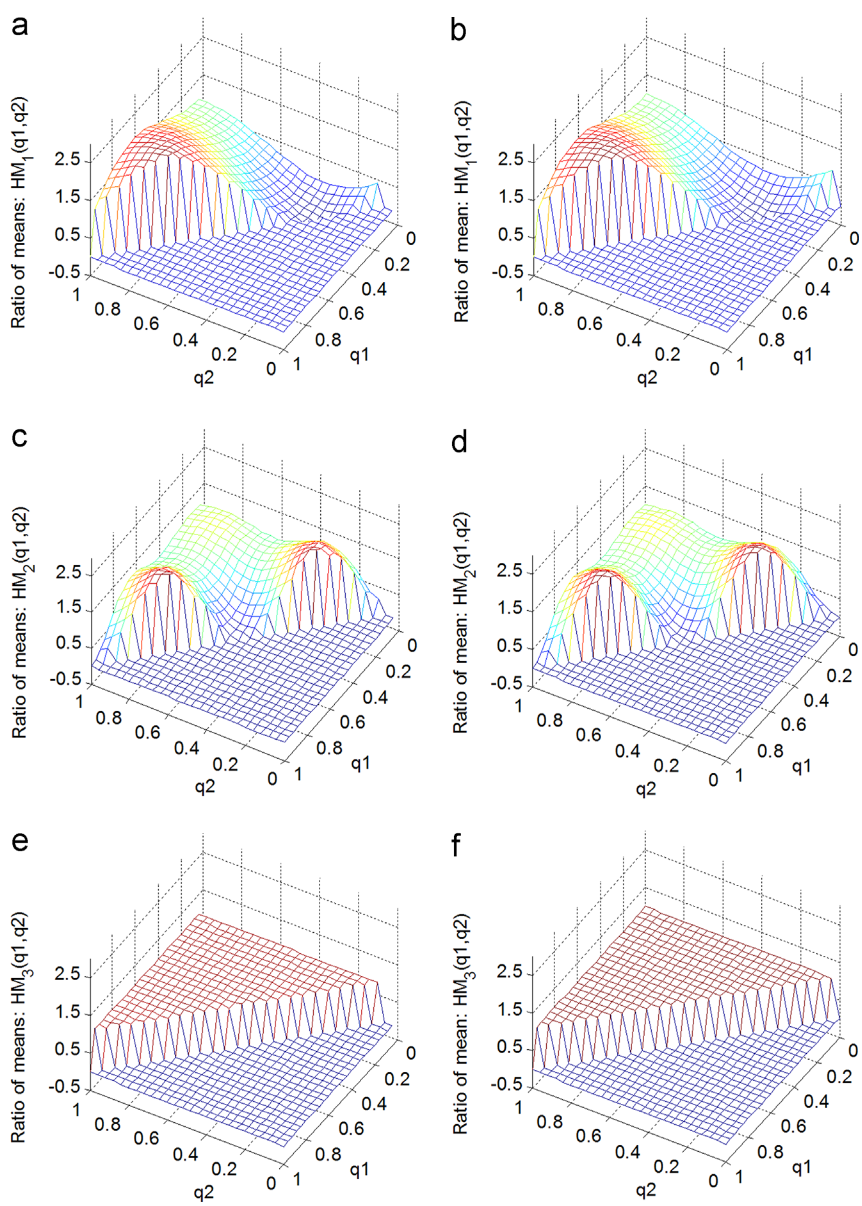

Fig. 1. 3D plots of the mean ratio functions of the Ishigami function, where (a), (c) and (e) denote the numerical estimates computed using 2000 samples, and (b), (d) and (f) stand for the analytical results.