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Performance analysis of two structured covariance matrix estimators in compound-Gaussian clutter

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Abstract

In this work we present a thorough performance analysis of two algorithms for estimating Toeplitz covariance matrices, the structured sample covariance matrix estimator (SCME) and the structured normalised SCME (NSCME), which are employed by adaptive radar detectors against Gaussian and compound-Gaussian clutter. Performance predictions are checked with real-life sea clutter data. © 2000 Elsevier Science B.V. All rights reserved.

Zusammenfassung

In dieser Arbeit wird eine sorgfältige Analyse der Leistungsfähigkeit zweier Schätzalgorithmen für Toeplitz-Kovarianzmatrizen präsentiert: den Schätzer für die aus den strukturierten Abtastwerten gewonnene Kovarianzmatrix (SCME) und den strukturierten normalisierten Schätzer SCME (NSCME), die bei adaptiven Radardetektoren gegen Gaußsche und verbundgaußsche Störungen eingesetzt werden Die vorhergesagten Leistungsfähigkeiten werden mit wirklichen Daten von Seeklutter verglichen. © 2000 Elsevier Science B.V. All rights reserved.

Résumé

Dans ce travail nous présentons une analyse complète des performances de deux algorithmes pour l'estimation des matrices de covariances de Toeplitz, de l'estimateur de matrices de covariances d'échantillons structurés (EMCS) et de EMCS normalisés structurés, qui sont employés par des détecteurs radars adaptatifs contre des entassements gaussiens et gaussiens composés. Les prédictions de performances sont vérifiées avec des données réelles d'entassements marins. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Parameter estimation; Coherent non-Gaussian clutter; Performance analysis

1. Introduction

Adaptive radar detection against Gaussian noise has been largely investigated in the past [4,7]. The same detection problem against a background of correlated compound-Gaussian clutter has been investigated only recently [1,5]. Different adaptive detection algorithms have been proposed to operate against Gaussian and compound-Gaussian clutter; most of them make use of secondary data from adjacent range cells to estimate the clutter covariance matrix, but the estimation algorithms

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are different. In a previous paper [6], the performance of the sample covariance matrix estimator (SCME) [7] and the *normalised* SCME (NSCME) [1] against compound-Gaussian clutter have been investigated. These two estimators furnish estimates that are positive-definite and Hermitian, but not Toeplitz. When the actual covariance matrix is Hermitian–Toeplitz, performance improvement can be obtained by incorporating this constraint into the detector formulation, as shown in [4,5]. In this paper we expand on [6] to consider the case of clutter covariance matrices which have the Toeplitz structure. To this purpose we proceed as follows. In Section 2, brief descriptions are provided for the clutter model as well as the covariance estimators. In Section 3, the expression for the mean square error (mse) of the structured SCME is derived and compared to that of the structured NSCME obtained by Monte Carlo simulation. We also checked our performance prediction with real-life sea clutter data. Some concluding remarks are given in Section 4.

2. Problem statement and estimators description

To estimate the clutter covariance matrix it is usually assumed that K blocks of signal-free secondary data, $\{\boldsymbol{z}_k\}_{k=1}^K$ from K adjacent range cells are available and that they are identically distributed (homogeneous clutter). According to the compound-Gaussian model, each element of the complex clutter vector $\boldsymbol{z}_k$ can be interpreted as the product of two independent random variables such that $\boldsymbol{z}_k = \sqrt{\tau_k}\boldsymbol{x}_k$ [8]. $\boldsymbol{x}_k \sim \mathscr{CN}(\boldsymbol{0}, \boldsymbol{M})$ is an $m \times 1$ complex Gaussian circular random vector, called the speckle, with normalised covariance matrix $\boldsymbol{M}$ (i.e., $[\boldsymbol{M}]_{ii} = 1$); τ_k is the texture and represents the local clutter power in the kth range cell. Given a specific value of τ_k, $\boldsymbol{z}_k$ is a complex Gaussian circular vector with conditional covariance matrix $E\{\boldsymbol{z}_k\boldsymbol{z}_k^{\mathrm{H}}|\tau_k\} = \tau_k\boldsymbol{M}$, with $E\{\cdot\}$ denoting statistical expectation. The unconditional clutter covariance matrix is $\boldsymbol{M}_Z = E\{\boldsymbol{z}_k\boldsymbol{z}_k^{\mathrm{H}}\} = \mu\,\boldsymbol{M}$, with $\mu = E\{\tau_k\}$. The speckle vectors $\{\boldsymbol{x}_k\}_{k=1}^K$ are assumed independent and identically distributed (IID), while the texture samples can be partially correlated with autocorrelation sequence $R_\tau[l] \triangleq E\{\tau_k\tau_{k+l}\}$, so $\{\boldsymbol{z}_k\}_{k=1}^K$ are orthogonal but not independent, save for the case of IID $\{\tau_k\}_{k=1}^K$, that implies $R_\tau[l] = E\{\tau_k^2\}\delta[l]$, where $\delta[\cdot]$ is the Kronecker's delta symbol. When matrix $\boldsymbol{M}$ is Hermitian–Toeplitz; i.e., it has identical elements on each diagonal, performance improvement can be obtained by incorporating this constraint into the detector formulation, as shown in [4,5]. Unfortunately, a closed-form expression of the maximum likelihood (ML) solution in such a case is not available, not even in Gaussian noise. In this case, we can incorporate the constraint by replacing the estimates on each diagonal with their sample average. Thus, the structured versions of SCME and NSCME are given by

$$\hat{m}_z[i, i+l] = [\hat{\boldsymbol{M}}_Z]_{i,i+l} = \frac{1}{K}\sum_{k=1}^{K}\left[\frac{1}{(m-l)}\sum_{n=1}^{m-l} z_k[n]z_k^*[n+l]\right] \tag{1}$$

and

$$\hat{m}[i, i+l] = [\hat{\boldsymbol{M}}]_{i,i+l} = \frac{1}{K}\sum_{k=1}^{K}\left[\frac{1}{(m-l)\hat{\tau}_k}\sum_{n=1}^{m-l} z_k[n]z_k^*[n+l]\right] \tag{2}$$

for $i = 1, 2, \ldots, m$, and $l = 0, 1, \ldots, m-1$, and $\hat{\tau}_k = (\boldsymbol{z}_k^{\mathrm{H}}\boldsymbol{z}_k)/m$ is the sample estimate of the clutter power in the kth secondary range cell. The contribution of this work is to analyse in detail the behaviour of the two structured estimators in (1) and (2) against compound-Gaussian clutter.

3. Performance analysis

Consider first the structured SCME in (1) and assume that the speckle sequences $\{x_k[i]\}$ are wide-sense stationary for each k. This assumption is reasonable in practice based on local stationarity within a given range cell. The estimator in (1) is unbiased regardless of the clutter amplitude probability density function (apdf); in fact,